2.4.13

13. Question Details

Find the constants a and b such that the function is continuous on the entire real line.

$$f(x)=\begin{cases} 9, & x\le -4 \\ ax+b, & -4<x<5 \\ -9, & x\ge 5 \end{cases}$$

a =

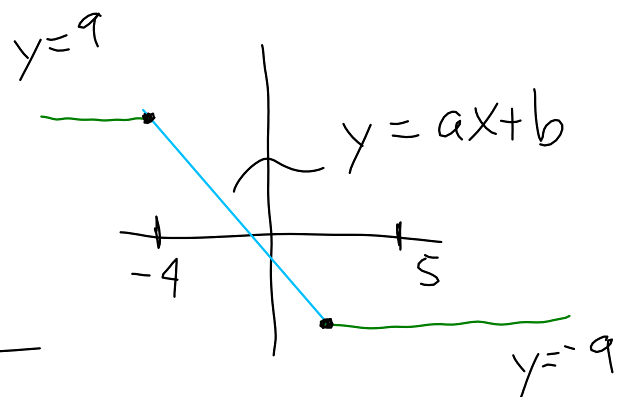
b =

f is cts at p

1) $\lim\limits_{x\to p} f(x)$ exists

2) $\lim\limits_{x\to p} f(x) = f(p)$

- $\lim\limits_{x\to -4^+} f(x) = 9$
- $\lim\limits_{x\to 5^-} f(x) = -9$



$$\begin{aligned} &a(-4)+b=9 \\ -(\ &a\cdot(5)+b=-9\ ) \end{aligned}$$

$-9a = 18 \longrightarrow \boxed{a=-2}$

$-2(-4)+b=9 \longrightarrow 8+b=9$

$\longrightarrow \boxed{b=1}$

$f(x)=-2x+1$

$f(-4)=-2(-4)+1=9$ ✓

$f(5)=-2(5)+1=-9$ ✓

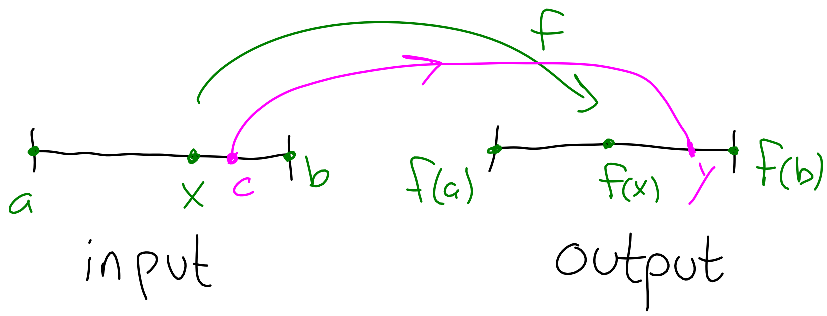
2.4.15

15. Question Details

Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of *c* guaranteed by the theorem.

$$f(x) = \frac{x^2 + x}{x - 1}, \quad \left[\frac{5}{2}, 4\right], \quad f(c) = 6$$

$c =$



$$f(x) = \frac{x^2+x}{x-1} \rightsquigarrow x=1?$$

$$\rightsquigarrow g(x) = x\frac{(x-1)}{(x-1)} = \frac{x^2-x}{x-1}$$

$$f(x) = \frac{x(x+1)}{x-1} \rightsquigarrow x=1 \text{ is a pt of discontinuity}$$

?. Is $1 \in [5/2, 4]$ ✓

$$f(c)=6 \to 6 = \frac{c(c+1)}{c-1} \to 6(c-1) = c(c+1)$$

$$\to 6(c-1) - c(c+1) = 0$$

$$\to -c^2 + 5c - 6 = 0$$

$\rightarrow c^2 - 5c + 6 = 0$

$\rightarrow (c-3)(c-2) = 0$

$\rightarrow \boxed{c=3}$, or $\cancel{c=2}$

which c is in $[5/2, 4]$

So $c=3$.

3.16 #4

4. Question Details

Find the derivative by the limit process.

$f(x) = \sqrt{x + 5}$

$f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\sqrt{x+h+5} - \sqrt{x+5}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{\left(\sqrt{x+h+5} - \sqrt{x+5}\right)\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}{\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}\right]$$

$$= \lim_{h \to 0} \cancel{\frac{1}{h}}\left[\frac{(\cancel{x}+h+\cancel{5}) - (\cancel{x}+\cancel{5})}{\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}\right]$$

$$= \lim_{h \to 0} \left[\frac{1}{\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}\right] = \boxed{1/2\sqrt{x+5}}$$

3.1a #1

1. Question Details

(a) Estimate the slope of the graph at points (x_1, y_1) and (x_2, y_2).

(x_1, y_1)

(x_2, y_2)

parallel lines have the same slope, so see if the line through (0,0) hits an integer lattice point (here, (2,2)).

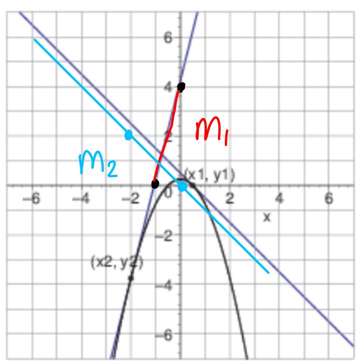
Since (0,0) and (2,2) are on the line

$$\rightarrow m_2 = \frac{\Delta y}{\Delta x} = \frac{2-0}{2-0} = 1$$

(b) Estimate the slope of the graph at points (x_1, y_1) and (x_2, y_2).

(x_1, y_1)

(x_2, y_2)



(-1,0) & (0,4) on line 1

$$\Rightarrow m_1 = \frac{\Delta y}{\Delta x} = 4/1 = \boxed{4}$$

(0,0) & (-2,2) on line 2

$$\Rightarrow m_2 = 2/-2 = \boxed{-1.}$$

3.1a #5

4. Question Details

Find the derivative by the limit process.

$$f(x) = \frac{1}{x-4}$$

$f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{1}{h}\left[f(x+h) - f(x)\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{1}{x+h-4} - \frac{1}{x-4}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{(\cancel{x}-\cancel{4})-(\cancel{x}+h-\cancel{4})}{(x+h-4)(x-4)}\right]$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left[\frac{\cancel{h}}{(x+h-4)(x-4)}\right]$$

$$= \lim_{h \to 0} \left[\frac{1}{(x+\cancel{h}-4)(x-4)}\right]$$

$$\boxed{= 1/(x-4)^2 .}$$

3.1a #5

5. Question Details

Find the derivative by the limit process.

$f(x) = \sqrt{x+7}$

$f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{1}{h}\left[\sqrt{x+h+7} - \sqrt{x+7}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\sqrt{x+h+7} - \sqrt{x+7}\overbrace{\left(\frac{\sqrt{x+h+7}+\sqrt{x+7}}{\sqrt{x+h+7}+\sqrt{x+7}}\right)}^{=1}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{(\cancel{x}+h+\cancel{7}) - (\cancel{x}+\cancel{7})}{\sqrt{x+h+7}+\sqrt{x+7}}\right]$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left[\frac{\cancel{h}}{\sqrt{x+h+7}+\sqrt{x+7}}\right]$$

$$= \lim_{h \to 0} \left[\frac{1}{\sqrt{x+\cancel{h}+7}+\sqrt{x+7}}\right] = \boxed{1/2\sqrt{x+7}.}$$