2.4.13

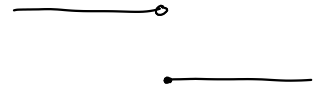
13. Question Details

Find the constants a and b such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} 7, & x \le -2 \\ ax + b, & -2 < x < 5 \\ -7, & x \ge 5 \end{cases}$$

$a =$

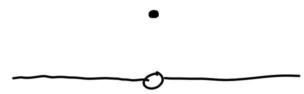
$b =$

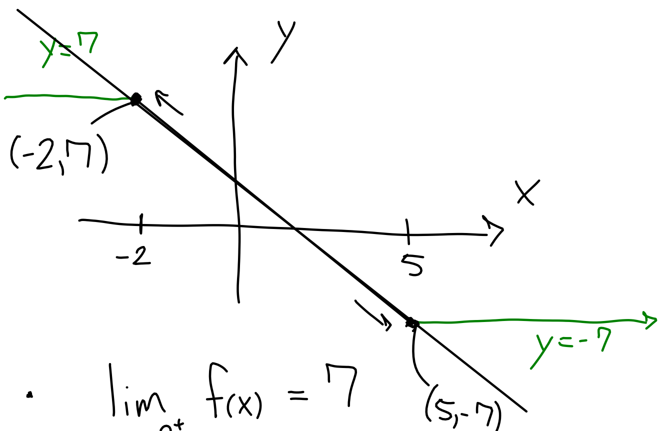


f is cts at p

1) $\lim_{x \to p} f(x)$ exist

2) $\lim_{x \to p} f(x) = f(p)$





- $\lim_{x \to -2^+} f(x) = 7$
- $\lim_{x \to 5^-} f(x) = -7$

$$a(-2) + b = 7$$
$$-\ (\,a(5) + b = -7\,)$$

$$-7a = 14 \to \boxed{a = -2}$$

$$(-2)(-2) + b = 7 \to 4 + b = 7 \to \underline{b = 3}$$

$$f(x) = -2x + 3$$

3.16 #4

4. **Question Details**

Find the derivative by the limit process.

$f(x) = \sqrt{x+1}$

$f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\left(\sqrt{x+h+1} - \sqrt{x+1}\right)\frac{\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}{\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}\right]$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left[\frac{(\cancel{x}+\cancel{h}+\cancel{1}) - (\cancel{x}+\cancel{1})}{\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}\right]$$

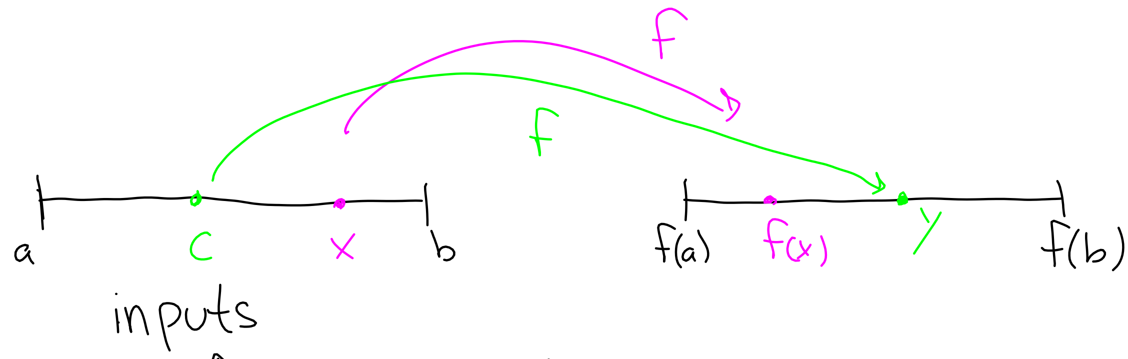
$$= \lim_{h \to 0}\left[\frac{1}{\left(\sqrt{x+\underset{\to 0}{h}+1} + \sqrt{x+1}\right)}\right]$$

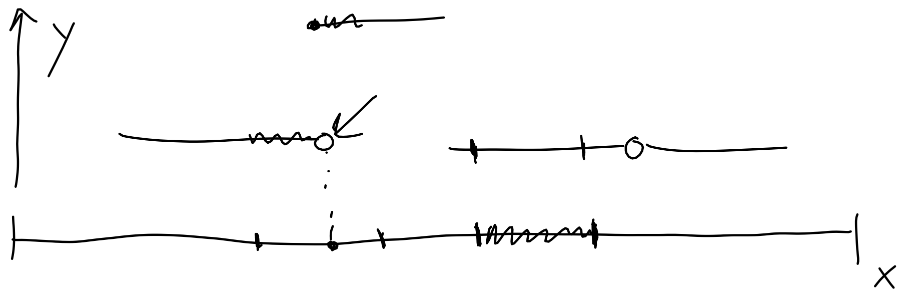
$$= \frac{1}{2\sqrt{x+1}}$$

5. Use the *Intermediate Value Theorem* (if applicable) to show that there is a solution on $[0, 1]$ to

$$x^5 + 2x = 1$$

Be sure to justify that you are allowed to use IVT by checking the necessary hypotheses.





$$x^5 + 2x = 1 \quad \text{iff} \quad x^5 + 2x - 1 = 0$$