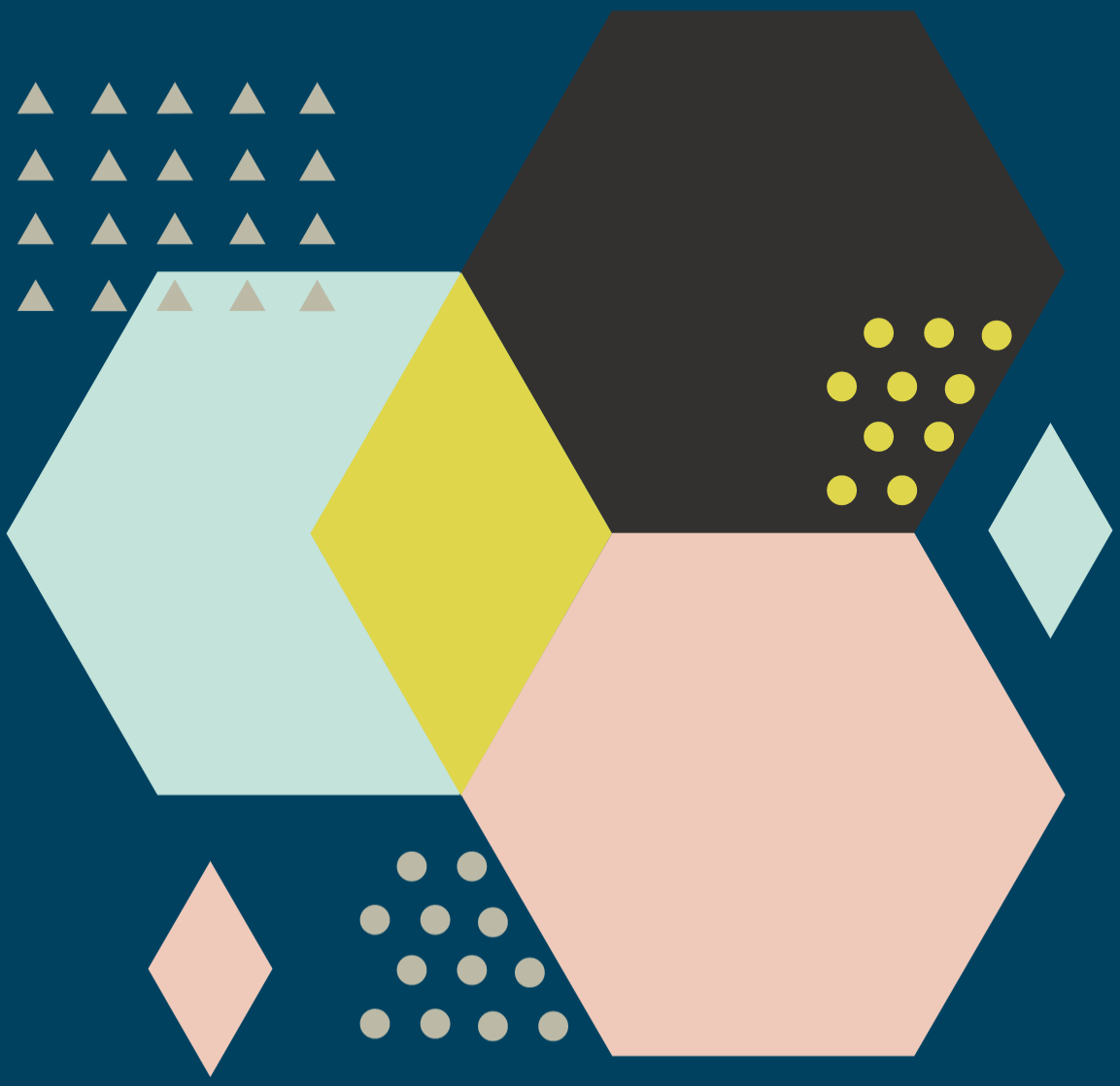
MOCK AMS 2021

UNIVERSITY OF GEORGIA



CONFERENCE BOOK

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About

About

The Mock AMS Conference is the capstone of our summer activities. It is intended to train our graduate students in communicating mathematics effectively. In particular, it trains them in the tricky art of giving a successful twenty minute general audience research talk.

Participating in Mock AMS Conference is intended to build a foundation for two most important talks that every mathematics PhD should be prepared to give: a talk about their research, and a talk on their favorite entertaining mathematics subject for an undergraduate audience.

More information about Mock AMS can be found on the website:
https://research.franklin.uga.edu/agant/mock-ams-conference-1

Useful Information

Talks will be held in **Room 304** of Boyd Graduate Studies, and simultaneously streamed on Zoom. To access the meeting link, please register here:

`https://zoom.us/meeting/register/tJIocu2vqD4vE9Mh0cUJ_pQi1qmfvUuS8a4q`

If you have any difficulties accessing the Zoom meeting, please reach out to the organizers.

Each day, there will be several blocks of 3 to 4 talks each, with 5-minute **breaks** between individual talks, and an extended 20-minute break between blocks. There will also be a longer lunch break in the middle of each day, see the schedule for more details.

Code of Conduct

Other attendees and speakers are your friends and colleagues, so please treat them in an accordingly respectful manner. In particular, please refrain from offensive or unwelcome comments/questions/discussions. This can include comments about ethnicity, national origin, race, gender, physical appearance, religion, age, body size, disability, veteran status, marital status, sexual orientation, gender identity, as well as anything deemed to be an insult, threat, discrimination, disruption, solicitation, or harassment of any kind. Attendees violating these guidelines may be sanctioned or expelled at the discretion of the organizers.

If you have any questions or would like to report any inappropriate behavior, please email any of the organizers directly.

Adapted from

- *https://conferences.math.mtu.edu/pint2021/coc/*
- *https://ccrem20.cms.math.ca/index.php/cms-code-of-conduct/*
- *http://ncngt.org/*

Organizing committee

If you have any questions, please feel free to reach out to the organizers via email:

D.Zack Garza Peter Woolfitt

Timetable

Tuesday, August 3rd

8:55 - 9:00	*Opening Remarks*	
9:00–9:20	**Matthew Just**	Racecars and avalanches, part II
9:25–9:45	**Han Lou**	Definitions of tangent vectors
9:50–10:10	**Patrick Akande**	Fibonacci–Eisenstein series and semi-modular forms
10:15–10:30	*Coffee/Tea Break*	
10:30–10:50	**Haiyang Wang**	An Introduction to Height Functions
10:55–11:15	**Komal Agrawal**	An introduction to the Erdos Multiplication Table Problem
11:20–11:40	**Amelia Ernst**	The intersection product on surfaces
11:40–1:00	*Lunch*	
1:00–1:20	**Aleksander Shmakov**	Single-Valued Elliptic Polylogarithms
1:25–1:45	**Skylar Zhang**	Construction of Besicovitch Sets
1:50–2:10	**Sarah Blackwell**	Distinguish Knotted Surfaces With This One Weird Trick
2:10–2:15	*Closing Remarks*	

Wednesday, August 4th

8:55 - 9:00	*Opening Remarks*	
9:00–9:20	**Nolan Schock**	Intersection theory in tropical geometry
9:25–9:45	**Alexander Tepper**	Percolation Theory and Critical Probabilities
9:50–10:10	**Jack Wagner**	A Quick Introduction to Highest Weight Categories
10:15–10:30	*Coffee/Tea Break*	
10:30–10:50	**Peter Woolfitt**	A Probabilistic Approach the the Weierstrass Approximation Theorem
10:55–11:15	**D. Zack Garza**	Computing Volumes of Moduli Spaces
11:20–11:40	**Freddy Saia**	Fields of moduli, rationality, and fake elliptic curves
11:40–1:00	*Lunch*	
1:00–1:20	**Arvind Suresh**	Realizing Galois representations in Jacobians of hyperelliptic curves
1:25–1:45	**Zhaiming Shen**	Kolmogorov Superposition Theorem
1:50–2:10	**Tyler Genao**	Galois representations under isogenies
2:10–2:15	*Closing Remarks*	

List of Abstracts – Talks

Agbolade Akande **Fibonacci–Eisenstein series and semi-modular forms**

In recent work, M. Just and R. Schneider defined a class of "semi-modular forms" on $\mathbb{C}$, in analogy with classical modular forms, that are "half modular" in a particular sense; and constructed families of such functions as Eisenstein-like series using symmetries related to integer partitions. Looking for further natural examples of semi-modular behavior, here we construct a family of Eisenstein-like series to produce semi-modular forms, using symmetries related to Fibonacci numbers instead of partitions. Generalized Fibonacci sequences yield analogous Eisenstein series, whose obstructions to semi-modularity are meromorphic functions depending on the initial terms.

Aleksander Shmakov **Single-Valued Elliptic Polylogarithms**

In forthcoming joint work with Daniel Litt, we develop a formalism for producing single-valued iterated integrals on algebraic varieties. The simplest example is the logarithm $\log(z)$ on $\mathbb{C}^\times$ whose single-valued avatar is $\log(z)$ More complicated examples include single-valued polylogarithms on $\mathbb{C} - \{0, 1\}$ whose single-valued avatars are the Bloch-Wigner-Zagier single-valued polylogarithms. We produce an elliptic analogue of the Bloch-Wigner-Zagier single-valued polylogarithms as single-valued iterated integrals of Eisenstein sums on once punctured complex elliptic curves and compare this to the previously ad-hoc constructions of these in the literature.

Alexander Tepper **Percolation Theory and Critical Probabilities**

Percolation theory describes the connectivity of random infinite graphs. These are frequently of the form $G = (\mathbb{Z}^d, E)$, where E is obtained by drawing edges between adjacent nodes in the integer lattice with fixed probability p. One is interested in the structure of a typical G as p is varied. Surprisingly, the connectivity properties of these graphs change dramatically around the so-called critical probability p_c. We will describe the basic objects and problems in percolation theory and prove the existence of a critical probability for the graphs described above.

Amelia Ernst **The intersection product on surfaces**

Algebraic surfaces are equipped with an intersection product that can be used to capture and describe a number of important geometric and topological properties. This talk will be an informal description of this product and some examples of it being applied to some computations.

Arvind Suresh **Realizing Galois representations in Jacobians of hyperelliptic curves**

We provide some results on the following problem: given a continuous representation ρ of the absolute Galois group G of $\mathbb{Q}$, construct an abelian variety A over $\mathbb{Q}$ (for example, an elliptic curve) which realizes ρ, i.e. such that ρ appears as a sub-representation of the natural Galois representation on the group of algebraic points of A. One natural reason this is interesting is because it gives information about how the rank of the abelian variety varies upon extension of the base field.

Some examples: suppose $K/\mathbb{Q}$ is a cyclic extension of degree 3 or 16; then we produce elliptic curves over Q whose ranks grow by 8 when passing from $\mathbb{Q}$ to K.

Freddy Saia **Fields of moduli, rationality, and fake elliptic curves**

This will be a mostly expository talk, beginning with a discussion of the field of moduli of an algebraic structure. We will consider the general question of how the field of moduli relates to fields of definition, and survey some results e.g. in the context of abelian varieties. Honing in on the lowest-dimensional cases, we will compare the cases of elliptic curves and certain abelian surfaces (maybe with additional structure, and I will give an idea of the questions I am working on and associated tools.

Haiyang Wang **An Introduction to Height Functions**

Height functions are used to quantify the complexity of solutions to Diophantine equations. They play an important role in studying rational points on algebraic varieties. In this talk, I will give a brief introduction to height theory, including height functions on projective spaces, Weil's height machine and the Néron-Tate height of elliptic curves. I will also discuss some applications of height functions.

Han Lou **Definitions of tangent vectors**

The notion of a tangent vector is one of the most important definitions in topology and geometry. It is the generalization of the definition of derivative in calculus to manifolds. In this talk, I will introduce three definitions of tangent vectors and prove they are equivalent.

Jack Wagner **A Quick Introduction to Highest Weight Categories**

In representation theory, we study groups or other algebraic objects by trying to understand how they can act on a vector space. In Lie theory, the algebraic structures we look at, such as reductive groups and Lie algebras, carry combinatorial structures with them. This helps us understand the relationships between representations using properties of an underlying partially ordered set called a weight space. All of this fits into the framework of a highest weight category, introduced by Cline-Parshall-Scott in 1988. Further, there are several important geometric examples of highest weight categories which can help us understand our algebraic objects better. In this talk, I will give motivation and define a highest weight category, and talk about a few examples and important tools this structure gives us.

Komal Agrawal **An introduction to the Erdos Multiplication Table Problem**

This problem consist of counting unique entries obtained by multiplying $1, 2, \cdots, N$ to $1, 2, \cdots, N$. In this talk I will discuss how probabilistic ideas help us to get an estimate of the order of magnitude of the count. Despite significant progress since Erdos we still don't have an asymptotic estimate for the count.

Matthew Just **Racecars and avalanches, part II**

An ordered integer factorization of n is a sequence of positive integers greater than 2 such that the product of these numbers is equal to n. A palindromic sequence (racecar) is a sequence that can be read the same way forwards and backwards, and an anti-palindromic sequence (avalanche) is a sequence with no mirror symmetry among its parts. Extending on recent work with George Andrews and Greg Simay, we give new results related to palindromic and anti-palindromic integer factorizations. This is joint work with Noah Lebowitz-Lockard.

Nolan Schock **Intersection theory in tropical geometry**

Tropical geometry can be thought of as a combinatorial version of algebraic geometry, where the most basic objects are polyhedral complexes in $\mathbb{R}^n$. In the nicest situation, these polyhedral complexes correspond to smooth toric varieties. I will describe an approach to intersection theory in tropical geometry based on the (well-known) intersection theory of toric varieties. Time permitting, I will explain some applications to understanding the intersection theory of certain algebraic varieties.

Peter Woolfitt **A Probabilistic Approach the the Weierstrass Approximation Theorem**

In this talk we will go over a surprising probabilistic approach to Weierstrass's useful approximation theorem.

Sarah Blackwell **Distinguish Knotted Surfaces With This One Weird Trick**

Algebraists hate it! Smoothly knotted surfaces cannot be distinguished by fundamental groups, but this One Weird Trick (Stallings folding) can be used to prove that they can be distinguished by group trisections. Perhaps regrettably for them, this is joint work with Rob Kirby, Michael Klug, Vince Longo, and Benjamin Ruppik.

Skylar Zhang **Construction of Besicovitch Set**

A Besicovitch set (also know as a *Kakeya set*) is a set of measure zero that contains line segments of all directions. In today's talk, we'll present Stein's construction of Besicovitch sets using triangles.

Tyler Genao **Galois representations under isogenies**

An elliptic curve E defined over a field F is afforded an action of the absolute Galois group $\mathrm{Gal}(\overline{F}/F)$ on its torsion subgroup $E[\mathrm{tors}]$, which gives rise to the various mod-N Galois representations of E. These Galois representations capture important information about the rationality of torsion points on E.

Elliptic curves which are isomorphic over F will have isomorphic Galois representations. In this talk, we'll explore what can be said about the Galois representations of *isogenous* elliptic curves, i.e., elliptic curves with a non-constant morphism between them (not necessarily injective).

D. Zack Garza **Computing Volumes of Moduli Spaces**

The moduli stack of Riemann surfaces $\mathcal{M}_{g,n}$ of genus g with n marked points can be equipped with a symplectic form, which can be wedged to its top power and integrated to produce a well-defined real number: the Weil-Petersson volume. Mirzakhani studied the asymptotic behavior in g and n of these volumes, deducing formulas that can be used to compute them all recursively, which often involve special values of $L-$functions.

In this talk, we'll compute the volume of $\mathcal{M}_{1,1}$, the moduli space of elliptic curves, in terms of the Dirichlet eta function $\eta(s)$.

Zhaiming Shen **Kolmogorov Superposition Theorem**

In this presentation, we will talk about Kolmogorov superposition theorem, which developed about half century after Hilbert raised his 13th problem in 1900 at the Second International Congress of Mathematicians in Paris. We will give a sketch of proof of the theorem based on the idea from Lorentz in 1962 and also present its application to function approximation and image reconstruction.