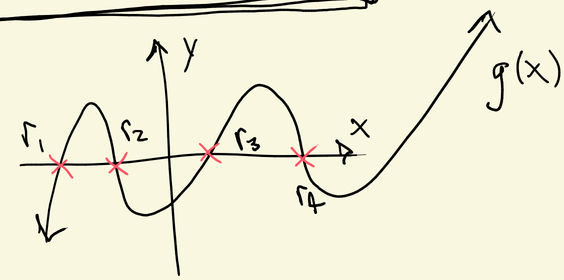
Quiz 5 (?) Review / Notes

① Find the roots to $\boxed{f(d) = (d-\alpha)(d-\beta)}$

Roots $\leftrightarrow$ $f(d) = 0$



$f(d) = d^2 - \alpha d - \beta d + \alpha\beta$

$\Rightarrow$ quadratic formula

$g(x) = (x - r_1)(x - r_2)(x - _3)(x - r_4)$

example $(ab = 0 \Rightarrow a = 0 \text{ or } b = 0)$

non-example $(ab = c \Rightarrow a = c \text{ or } b = c)$

Three things to "aim for"

1) $g(x) = (x - r_1)(x - r_2)(x - r_3)(x - r_4) = 0$

$x - r_1 = 0$ or $x - r_2 = 0$ or ...

ex $g(x) = (x-2)(x-3)(x-12)(x+4)$

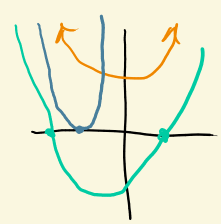
$\underline{a=3, b=4} \Rightarrow 3 \cdot 4 = \underline{12 = c}$

2) $g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

ex $g(x) = 7x^3 + 4x^2 + x - 10$

3) $g(x) = (ax^2 + bx + c)($ ~~~ $)$ other stuff of forms 1 or 2

multiplies to $x^2 + \dots$



Solution

$\boxed{f(x) = (x-\alpha)(x-\beta) = 0}$

input

$\Rightarrow d - \alpha = 0$ or $d - \beta = 0$

$\Rightarrow \underline{d = \alpha \text{ or } d = \beta}$

$\boxed{\Rightarrow \text{roots}(f) = \{\alpha, \beta\}}$

How to check?

? check ?

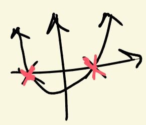
$$\text{roots}(f) = \{\alpha, \beta\} \implies \underbrace{\begin{matrix} f(\alpha) = 0 & \checkmark \\ f(\beta) = 0 & \checkmark \end{matrix}}$$

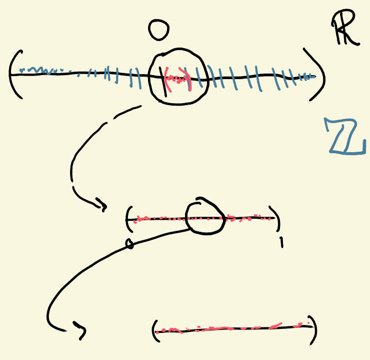
$$\implies f(\alpha) = \underbrace{(\alpha - \alpha)}_{0}\underbrace{(\alpha - \beta)}_{?}$$
$$= 0 \cdot ?$$
$$= 0$$

$$\implies f(\beta) = (\beta - \alpha)(\beta - \beta)$$
$$= ? \cdot 0$$
$$= 0$$

Evidence vs proof

Evidence	Proof
• Graphs	• Algebra!
• Tables	• Definitions

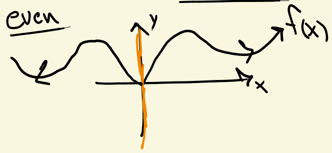




Mnemonic

Definitions

even



$f(-x) = f(x)$

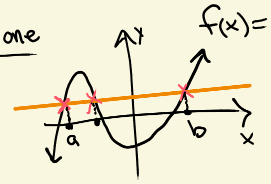
• Calculators

• Mnemonics

(symmetry of graph)

one-to-one

$f(x) = ?$ equation



$$f(a) = f(b) \Downarrow a = b$$

Note on Optimization Problems

Be systematic!!

1) (Always) Draw a schematic picture, name thing

2) Declare your objective function (Ex $\boxed{A(w,h) = w \cdot h}$)

3) Declare your constraint function (Ex $F(w,h) = 2w+2h$, $F(w,h) = 100$ ft)

4) Solve constraint for one variable, then substitute into objective

$h = g(w)$ $\qquad$ $A(w, h) = A(w, g(w))$] one variable

$2w+2h=100$

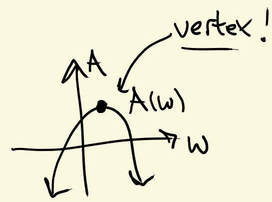
$h = g(w) = \dfrac{100-2w}{2}$

$A(w, g(w)) = w \cdot g(w)$

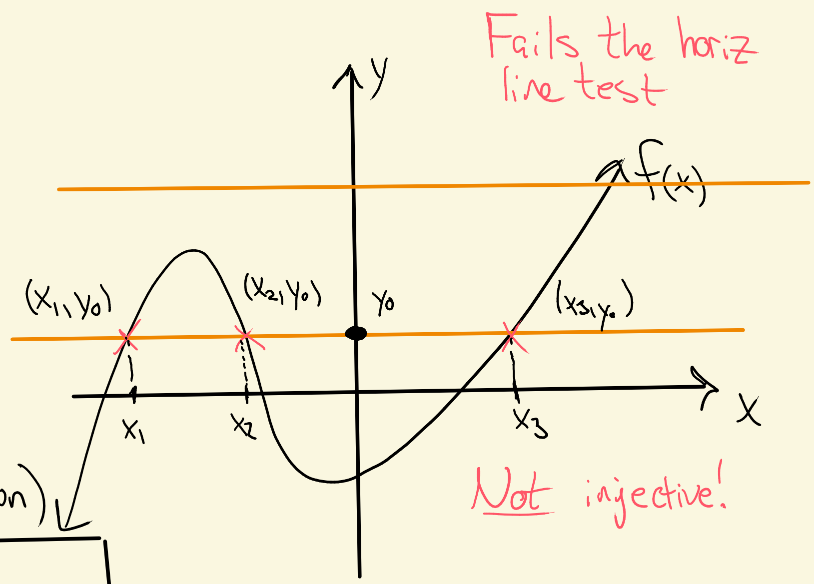
$= w\left(\dfrac{100-2w}{2}\right)$] only w appears!

5) Apply optimization tools

(Ex: vertex of a parabola



Injectivity / "One-to-one"



Mnemonic: Horizontal line test

f is injective (one-to-one) iff f passes the horiz. line test

(passes means only 1 intersection)

Def: f is injective iff

$$f(x_1) = f(x_2) \quad \cdots \quad \Rightarrow x_1 = x_2$$

Slogan: Every $\underbrace{\text{output}}_{y_0,\, f(x_1),\, f(x_2)}$ only comes from one unique $\underbrace{\text{input}}_{x_1,\, x_2}$

⚠ Easy to confuse with the vertical line test!
(Checks if f is a function)

Slogan: Every input gets mapped to a unique output