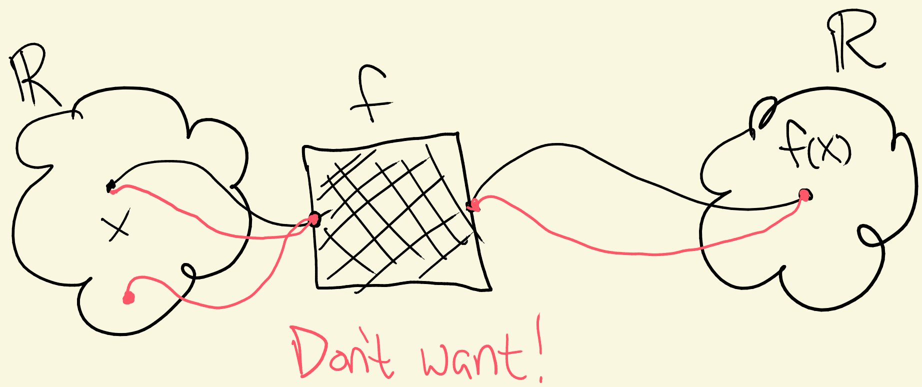
Quiz - Review

Preclasses & Quizzes

- Scratch work vs presenting a solution
- Why care about injectivity?

> **Def:** A function $f: \mathbb{R} \to \mathbb{R}$ is injective iff
> $(f(a) = f(b) \Rightarrow a = b.)$



Proof	evidence
Definition	Horizontal line test

Ex: Define $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = 3x + 2$$

Q: Is f injective?

- Assume $f(x_1) = f(x_2)$

$\Rightarrow \quad 3x_1 + 2 = 3x_2 + 2$

$\Rightarrow \quad 3x_1 = 3x_2 \qquad$ (Subtracting 2)

$\Rightarrow x_1 = x_2,$ (dividing by 3)

$\boxed{\text{So } f \text{ \underline{is} injective.}}$

Why care?. Non-example: $g : \mathbb{R} \to \mathbb{R}$

$g(x) = x^2$

Q: Is g injective?

Imagine doing a problem

$\vdots$

$g(a) = g(b)$

$\vdots$

$a = b$

$\Rightarrow \boxed{a^2 = b^2}$

$\cancel{\Rightarrow} \sqrt{a^2} = \sqrt{b^2}$ (Taking sq. roots)

$\cancel{\Rightarrow} \boxed{a = b.}$ $\blacksquare$

Counterexample

$a = -2, \ b = 2$

so $a \neq b$, but

$a^2 = (-2)^2 = 4$

$b^2 = (2)^2 = 4$

Why?. $\sqrt{a^2} = |a| = \pm a$

$\sqrt{a^2} = \sqrt{b^2}$

$\Rightarrow |a| = |b|$

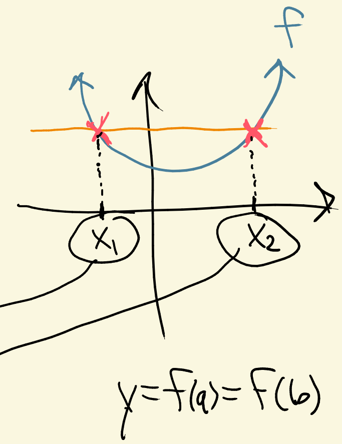
$\Rightarrow a = \pm b$

· Showing a function f is <u>not</u> injective

$$\text{Not}\ \big(f(a)=f(b) \Rightarrow a=b\big) \equiv$$

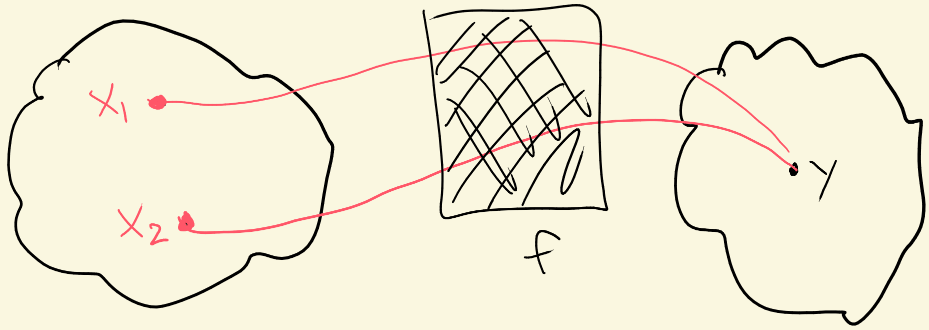
Prove f is not injective:

There exists a pair a, b with $\boxed{f(a)=f(b)}$ <u>but</u> $\boxed{a \neq b.}$



$x_1 = a$

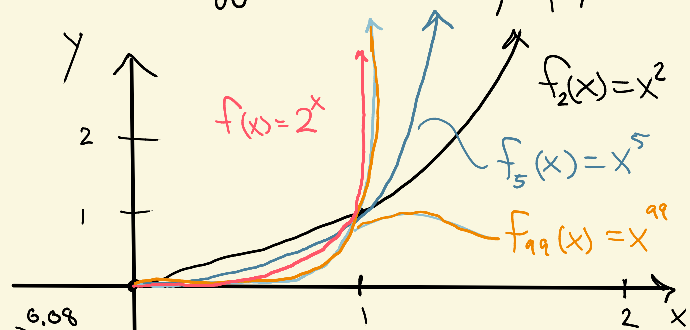
$x_2 = b$

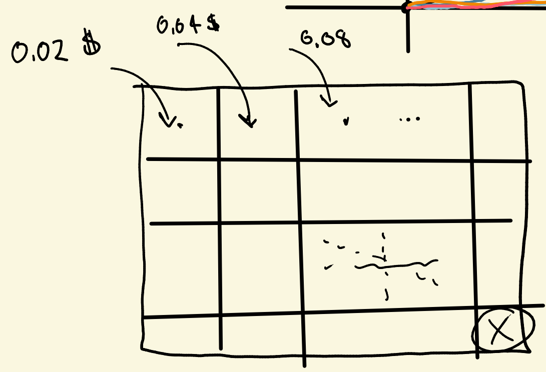


Break!

Exponential Formulas

Important Aside: Exponential growth is "eventually bigger" than any polynomial





64 tiles

→ how much is on tile 64?

Applications: Compounded Interest

1) Discrete

$$\boxed{A_D(t) = P_0\left(1+\frac{r}{n}\right)^{nt}}$$

- P_0: Initial value $(t=0)$
- r: Rate $(0 \le r \le 1)$ For interest!
- n: Number of events per unit time

Rate of $6\% \Rightarrow r = \frac{6}{100}$

⚠ Common mistake: $r=6$

2) Continuous

$$\boxed{A_c(\underline{\underline{t}})} = \text{"}\lim_{n\to\infty} A_D(t)\text{"}$$

$$\boxed{= P_0 e^{rt}}$$

- P_0: $\overbrace{\text{Initial Amt}}^{A(0)}$ $(t=0)$
- r: Rate

Interest: $0 \le r \le 1$