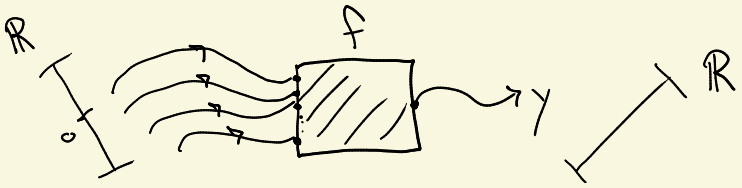
Tuesday

Setup: Given a model

$$f: \mathbb{R}^n \to \mathbb{R}$$

$$f(x_1, x_2, \cdots, x_n) = y$$

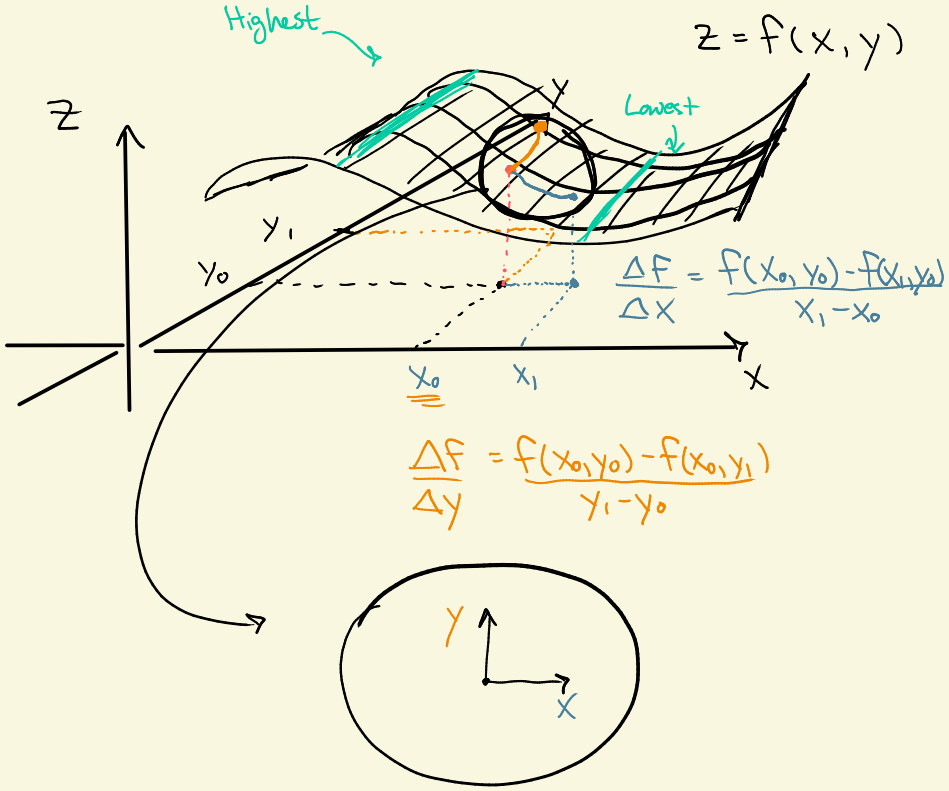


Question: Which x_i has the most impact?

Goal: Analyse this question.

Consider the quantities

$$\text{"}\frac{\partial f}{\partial x_i}\text{"} = \text{"}\frac{df}{dx_i}\text{"} = \frac{\Delta f}{\Delta x_i}$$



Example

Function of interest: $A(m,t) = m\,e^{c(t-270)}$

"Constraint" : $P(m,t) = 3m^2\,e^{-b(t-270)}$

Step 1: Solve for m after setting

A = some #

P = some #

- $A = m\,e^{c(t-270)}$

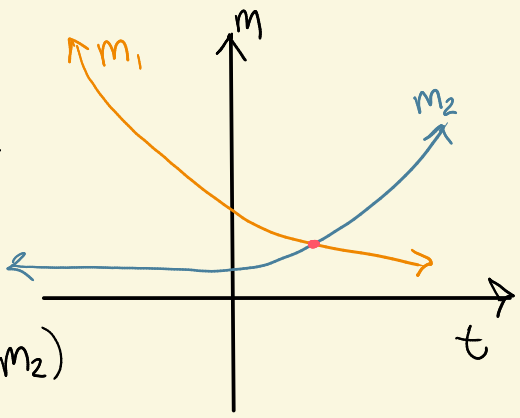
$$\Rightarrow \boxed{m = A e^{-c(t-270)}}$$

$$P = 3m^2\,e^{-b(t-270)}$$

$$\Rightarrow \boxed{m = \left(\tfrac{1}{3} P e^{b(t-270)}\right)^{\frac{1}{2}}} \neq \pm\sqrt{\tfrac{P}{3}}\; e^{b(t-270)/2}$$

$$m_1(t) = A e^{-c(t-270)}$$

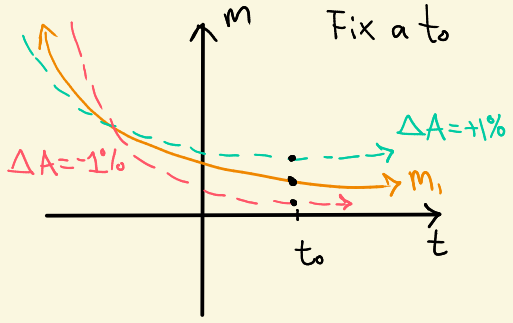
$$m_2(t) = \left(\tfrac{1}{3} P e^{b(t-270)}\right)^{\frac{1}{2}}$$



Two parallel analyses (on m_1, m_2)

m_1 depends on: A, c, t

ΔA	Δm_1
$\pm 1\%$	$\pm 2\%$
⋮	⋮



Same thing with c

Δc	Δm_1
$\pm 1\%$	$\pm ?\%$
⋮	⋮

(Maybe also with t)

Repeat with m_2

· Compare graphs ⇝ What's the story?

Part 2

$$m_1(t) = Ae^{-c(t-270)}$$

$$m_2(t) = \left(\tfrac{1}{3}Pe^{b(t-270)}\right)^{\frac{1}{2}}$$

Set $m_1(t) = m_2(t)$ (Intersection)

$$\Rightarrow Ae^{-c(t-270)} = \left(\tfrac{1}{3}Pe^{b(t-270)}\right)^{\frac{1}{2}}$$ solve for t

Math goggles: $e^{-2ct} \times e^{bt}$

$$\Rightarrow t = (540c + 270b + 2\ln(\sqrt{\tfrac{3}{P}}A))(2c+b)^{-1}$$

* Solving this: we'll cover this in-class

Depends on: $\{A, P, c, b\}$

Similar analysis of $\frac{\Delta t}{\Delta A}$, $\frac{\Delta t}{\Delta P}$, etc

ΔA	Δt
$\pm 1\%$	$\pm ?\%$

ΔP	Δt

+ Graphs if they help tell your story!