Tuesday Oct 27th

- Quick note: Modifying in-place & Scratch work

↳ Do scratch work sep.

Ex Solve for x in

$$\log_6(x+1) + \log_6(x-1) = 1/3$$

Scratch →

$$\cancel{6}^{\log_6(x+1)} \quad \cancel{6}^{\log_6(x-1)} = 6^{\frac{1}{3}}$$

$$(x+1)(x-1) = 6^{\frac{1}{3}}$$

$$x^2 - 1 = 6^{1/3}$$

$$+1 \qquad +1$$

$$\sqrt{x^2} = \sqrt{6^{1/3}+1}$$

$$x = \pm\sqrt{6^{1/3}+1}$$

Soln

$$\log_6(x+1) + \log_6(x-1) = 1/3$$

$$\Rightarrow (x+1)(x-1) = 6^{1/3} \qquad \text{(taking logs)}$$

$$\Rightarrow x^2 - 1 = 6^{1/3}$$

$$\boxed{\Rightarrow x = \pm\sqrt{6^{1/3}}}$$

Logs: Why care?!

1) Humans are very bad with large numbers

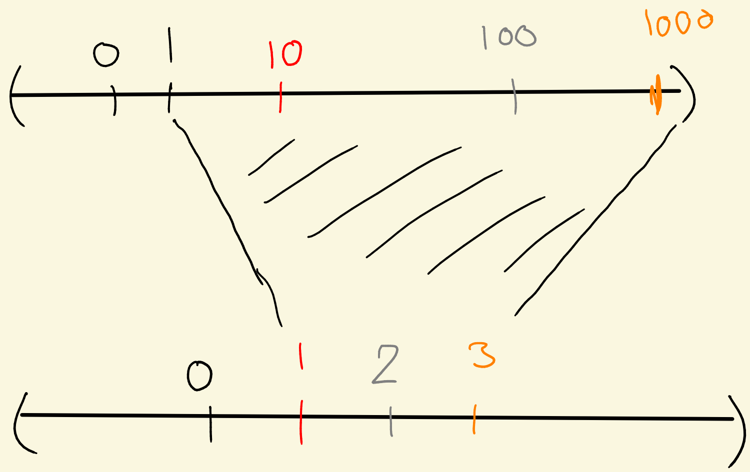
Where is 1 million?

2018 US Budget: 1.068 trillion

But with logs

$$\left.\begin{aligned} 1\,M &= 10^{6} \\ 1\,B &= 10^{9} \\ 1\,T &= 10^{12} \end{aligned}\right\} \xrightarrow{\log_{10}} \begin{matrix} 6 \\ 9 \\ 12 \end{matrix}$$

"Telescope"

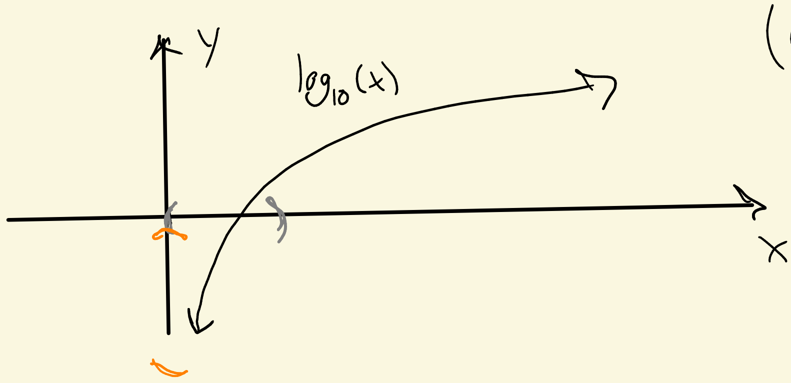


$\log_{10}(\cdot)$

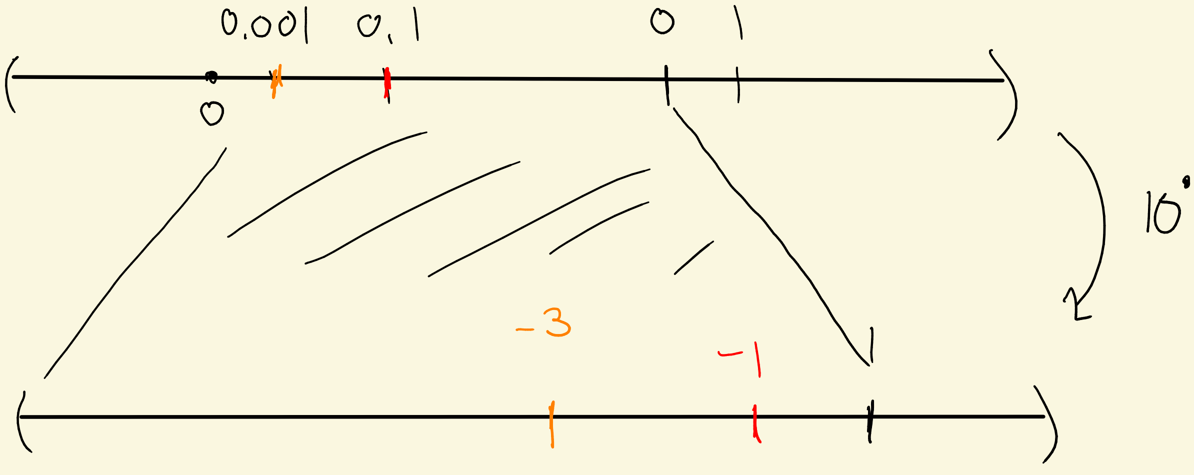
$(1, \infty)$

$\downarrow$

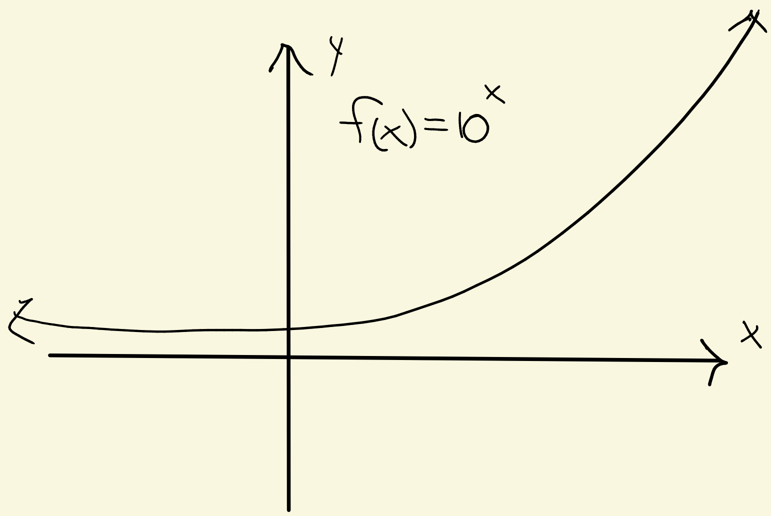
$(0, \infty)$ "squashed"



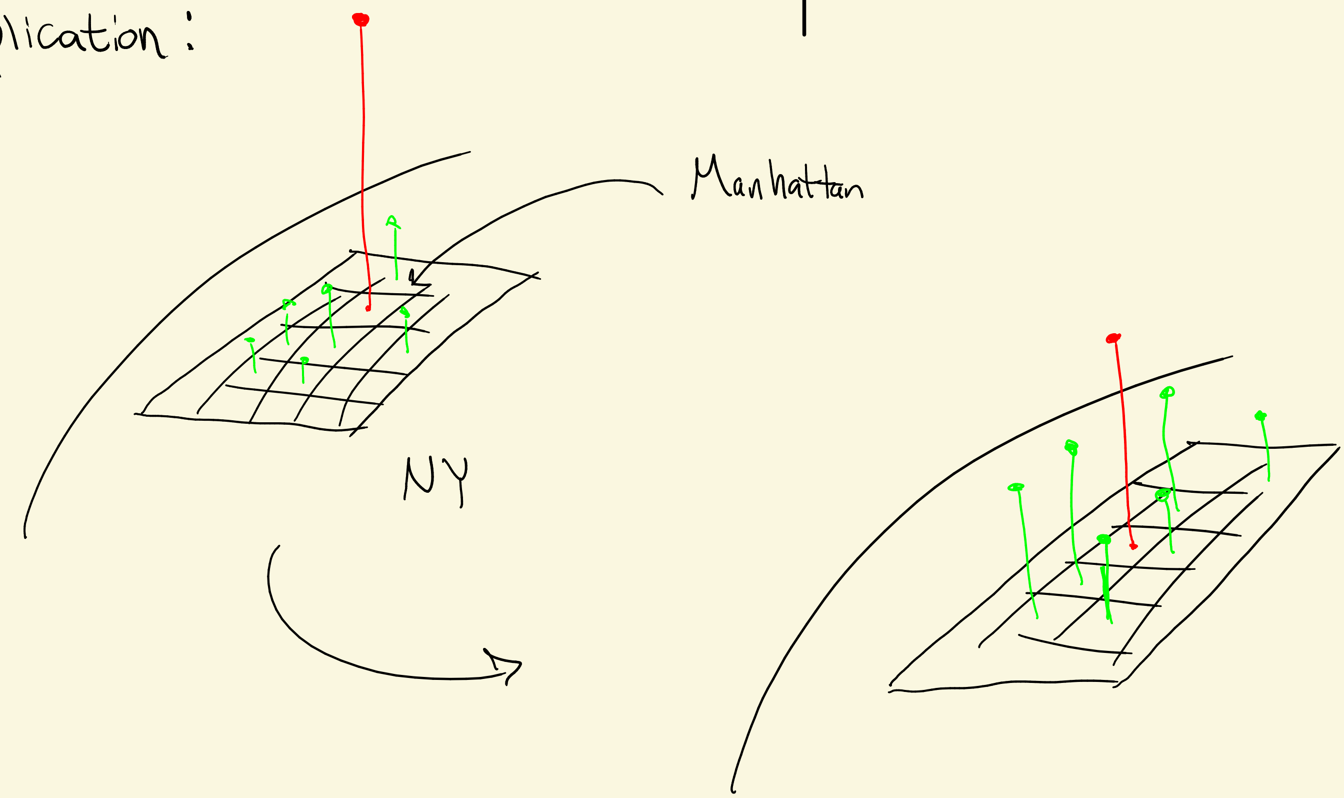
Exponentiation : "microscope"



$(0,1) \mapsto (-\infty, 0)$



Real-world Application:



Use: multiplying large #s

Eg: Compute $129 \cdot 505$.

$\log_2(129) \approx 7$

$\log_2(505) \approx 9$

Look up in a table

$$\log_2(129 \cdot 505) = \log_2(129) + \log_2(505)$$

$$\approx 7 + 9$$

$$\Rightarrow \log_2(129 \cdot 505) \approx 16$$

$$129 \cdot 505 \approx 2^{16} = 65{,}536$$

Review properties

$\log_{?}(?) = 1 \longrightarrow \log_\beta(\beta) = 1$

$\log_\beta(\beta) = \gamma \Rightarrow \beta^{\log_\beta(\beta)} = \beta^\gamma$

$\Rightarrow \beta = \beta^\gamma$

$\Rightarrow \gamma = 1$

$\ln(e) = 1$

$\log_{10}(10) = 1$

$\ln(e^{f(x)}) = f(x)\,\underbrace{\cancel{\ln(e)}}_{}^{1}$

$= f(x).$

① $\alpha^{\log_\alpha(?)} = ?$ $\alpha^{\log_\alpha(f(x))} = f(x)$

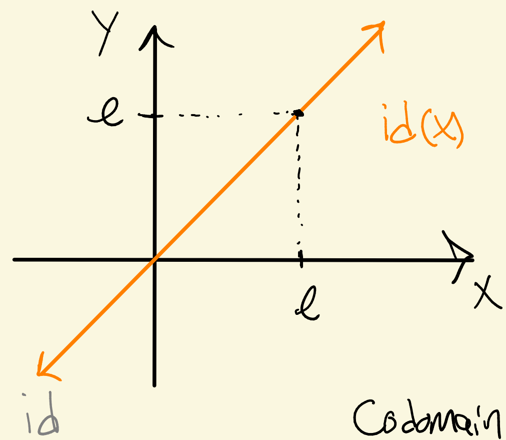
② $\log_\alpha(\alpha^{?}) = ?$ $\log_\alpha(\alpha^{f(x)}) = f(x)$

Should think of these as <u>functions</u>

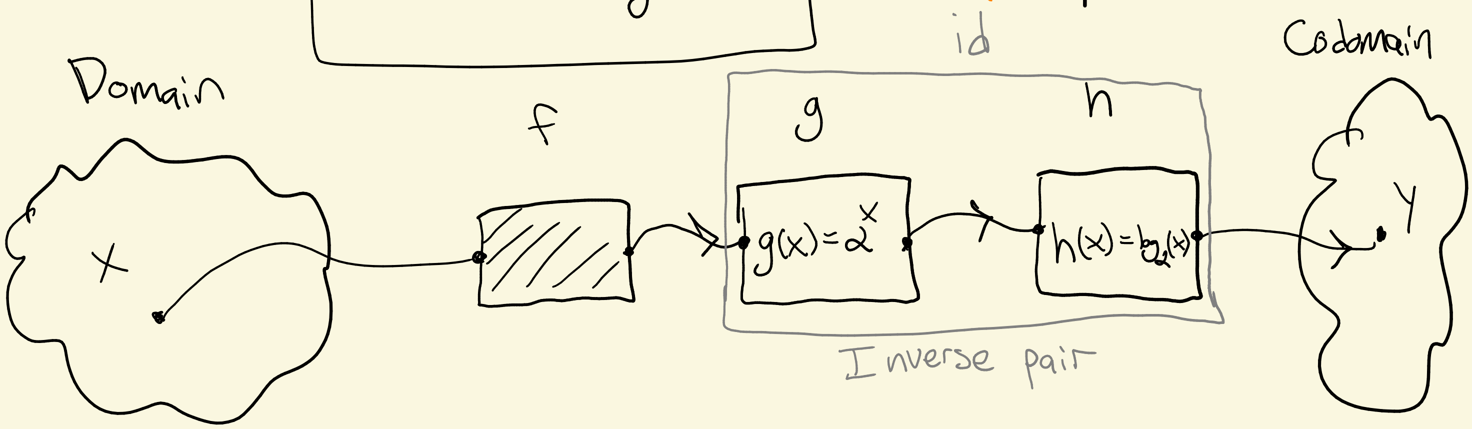
$g(x) = \alpha^x$, $h(x) = \log_\alpha(x)$

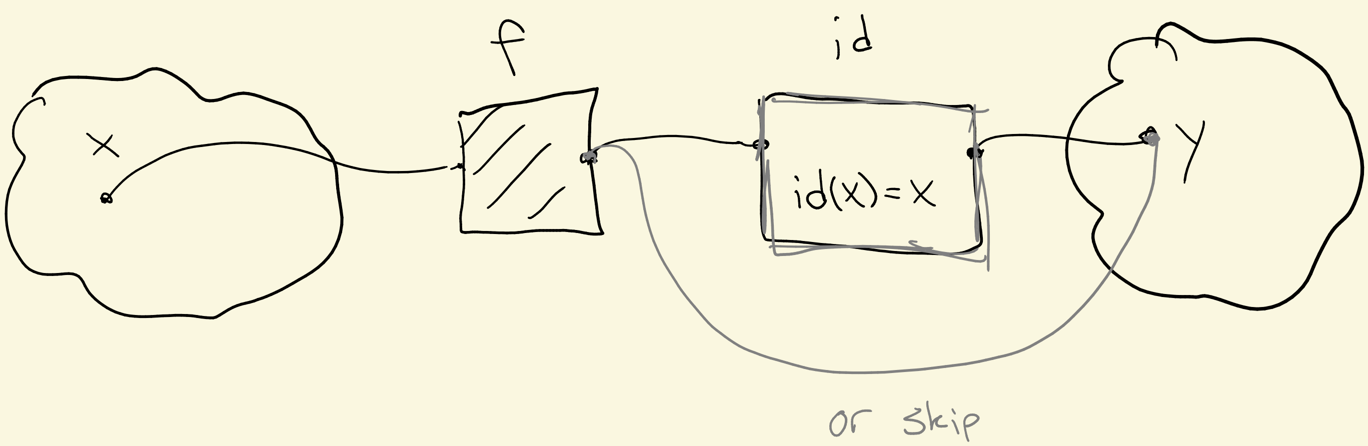
$\underbrace{(g \circ h)}_{id}(x) = x = id(x)$ $\underbrace{(h \circ g)}_{id}(x) = \underline{x} = id(x)$

$id(x) = x$



$\boxed{\text{Consider } f \circ g \circ h}$





Note : Trig functions will also have inverses

$$f(x) = e^x \ , \ g(x) = \ln(x) \Big\} \ \text{"} g \equiv f^{-1} \text{"}$$
$$\text{"} f \equiv g^{-1} \text{"}$$

⚠

$3^{-1} = 1/3$, $x^{-1} = 1/x$, $\sin^{-1}(x) \neq 1/\sin(x)$

arcsin(x) = inverse function of sin(x)

$\tan^{-1}(x)$ = inverse function for tan(x)

$\neq 1/\tan(x) = \cot(x)$

$= \arctan(x)$

$$\tan(\tan^{-1}(x)) = id(x) = x$$
$$\tan^{-1}(\tan(x)) = id(x) = x$$

Word Problems

Exponential change : Grows very fast : For any n

Claim $e^x > x^n$ eventually (for $x >> 0$)

Precise statement 1) $\frac{e^x}{x^n} \xrightarrow{\lim x\to\infty} \infty$

2) $\frac{x^n}{e^x} \xrightarrow{\lim x\to\infty} 0$

$f(t) = P e^{rt}$ $\begin{cases} r<0 & \text{decay} \\ r=0 & \text{constant} \\ r>0 & \text{growth} \end{cases}$

- Can always find P if we know initial conditions

$f(0) = P_0 \Rightarrow f(0) = P$

$f(0) = P e^{r\cdot 0} = P e^0 = P$