Quiz: Sections 3.5A, 3.5B, 3.6

- Solving exponential equations (also logs)

Ex Let $y = 3e^{x+2} - 3$, solve for x.

$\ln\left(\frac{y+3}{3}\right) - 2 = x$

scratch work

$$y = 3e^{x+2} - 3$$

$$\Rightarrow \quad y+3 = 3e^{x+2}$$

$$\Rightarrow \quad \frac{y+3}{3} = e^{x+2}$$

$$\Rightarrow \ln\left(\frac{y+3}{3}\right) = \ln\left(e^{(x+2)}\right)$$

Inverse pair

$f(x) = e^x$

$g(x) = \ln(x)$

$\ln(e^{h(x)}) = h(x)$

$$\Rightarrow \ln\left(\frac{y+3}{3}\right) = (x+2)\ln(e)$$

$\ln(e) = 1$

$\Rightarrow \ln\left(\frac{y+3}{3}\right) - 2 = x$.

or $\ln(y+3) - \ln(3) - 2 = x$

Ex $y = \log_{10}(5x+2)$, solve for x

scratch

$\frac{10^{y-2}}{5} = x$

$y = \log_{10}(5x+2)$

$\Rightarrow 10^y = 10^{\log_{10}(5x+2)}$

$\Rightarrow 10^y = 5x+2$

$\Rightarrow \boxed{\frac{10^y - 2}{5} = x}$

Using $f(x) = \log_{10}(x)$

$g(x) = 10^x$

are an inverse pair

Word Problems

- Exponential change (growth/decay)

$$P(t) = P_0 e^{rt}$$] given r

- Given initial conditions (i.e. $P(0) = \alpha$), then you can solve for P_0 (number)

$$\alpha = P(0) = P_0 e^{r \cdot 0} = P_0 \cdot 1$$

$$\Rightarrow f(t) = \alpha e^{rt}$$

Ex $P(t) = P_0 e^{-3t+1}$, $P(0) = 13$

$$13 = P(0) = P_0 e^{-3\cdot 0 + 1} = P_0 \cdot e^1$$

$\Rightarrow P_0 = 13/e = 13e^{-1}$ (Note: $P_0 \neq 13$)

- We can solve with proportions even when P_0 is not known

Eg $A(t) = A_0 e^{5t-3}$, when does $A(t)$ double?

Eg $A(t) = A_0 e^{5t}$, when does $A(t)$ double?

$\Rightarrow$ Solve for t such that

($A_0 \neq A(0)$ necessarily!)

$$A(t) = 2 \cdot A_0$$

$$2 \cdot A_0 = A(t) = A_0 e^{5t}$$

$$\Rightarrow 2 \cdot A_0 = A_0 e^{5t}$$

$$\Rightarrow 2 = e^{5t}$$

$$\Rightarrow \ln(2) = 5t$$

$$\Rightarrow (1/5)\ln(2) = t.$$

Quiz stuff ↑

Trigonometry

⚠ Always draw a picture!

- Radians

→ Always convert (before calculating) from degrees

Eg Better to leave solutions in the form

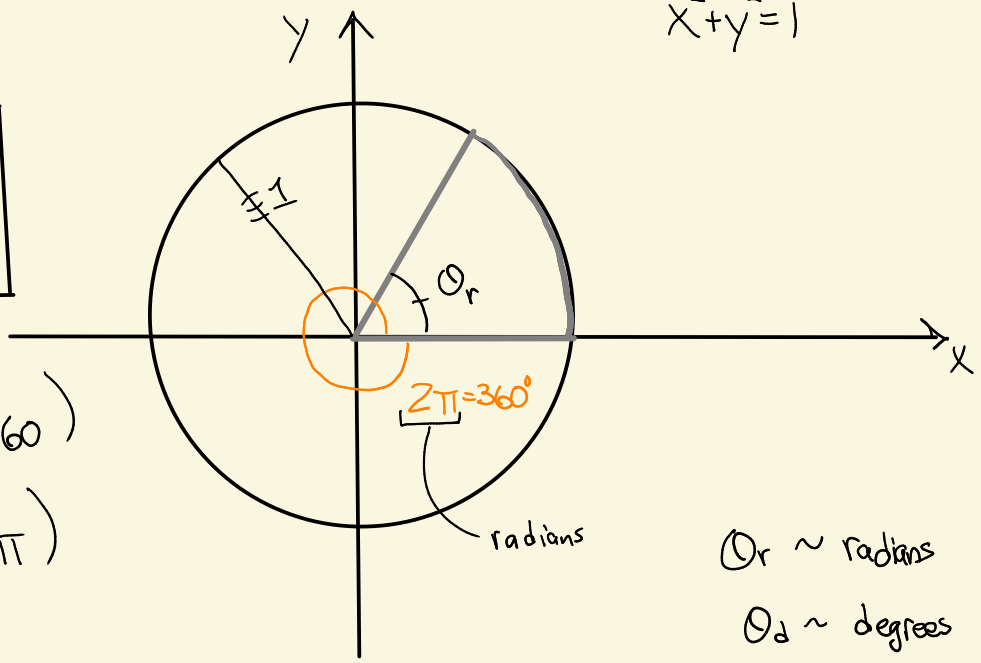
$\sin(10\pi/3)$ vs 1.2178.... (numerical approx. ≈)

Important equation

$$\frac{\theta_r}{2\pi} = \frac{\theta_d}{360}$$

→ $\theta_r = 2\pi\left(\theta_d/360\right)$

→ $\theta_d = 360\left(\theta_r/2\pi\right)$

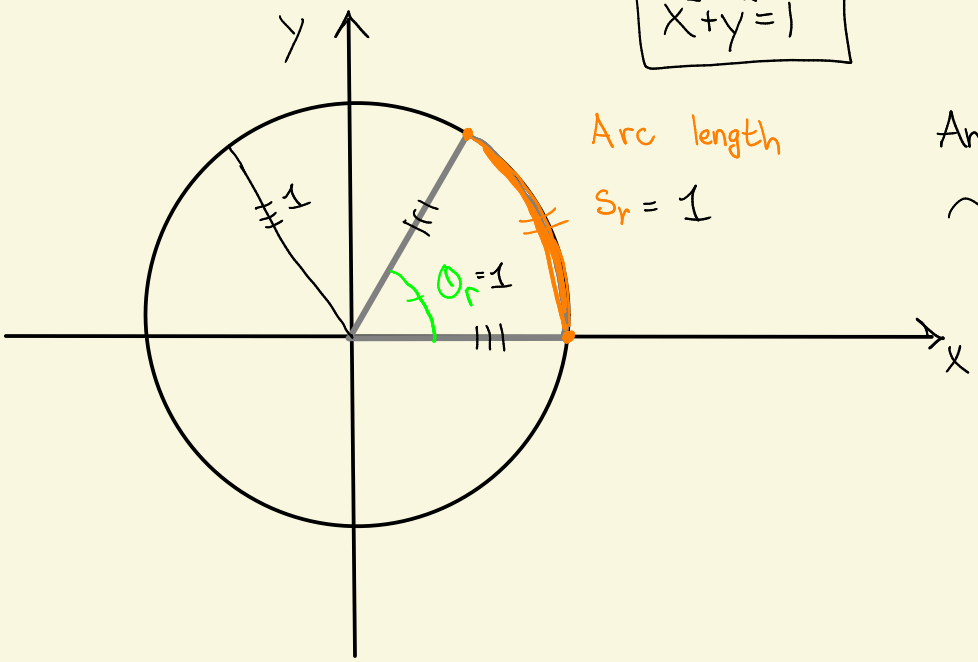


$x^2+y^2=1$

$\theta_r \sim$ radians

$\theta_d \sim$ degrees

$x^2+y^2=1$



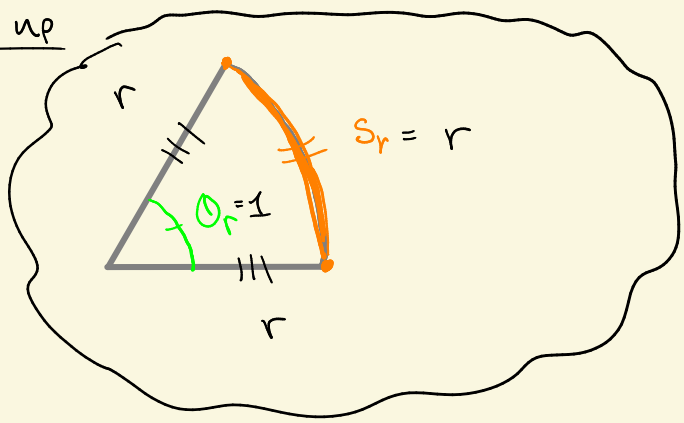
Arc length

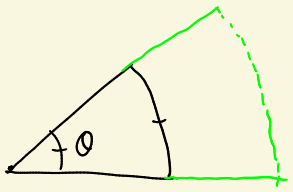
$s_r = 1$

Angle of 1 radian

↝ Arc length of 1 unit

Scale up





↪ Upshot: $s_r \propto \theta_r$ (Proportional)

s_r: Arc length

θ_r: Angle in radians

Important Review

- Area of a circle of radius r:

$$\text{Area}(r) = \pi \cdot r^2.$$

- Circumference of a circle of radius r:

$$\text{Circumference}(r) = 2\pi r.$$