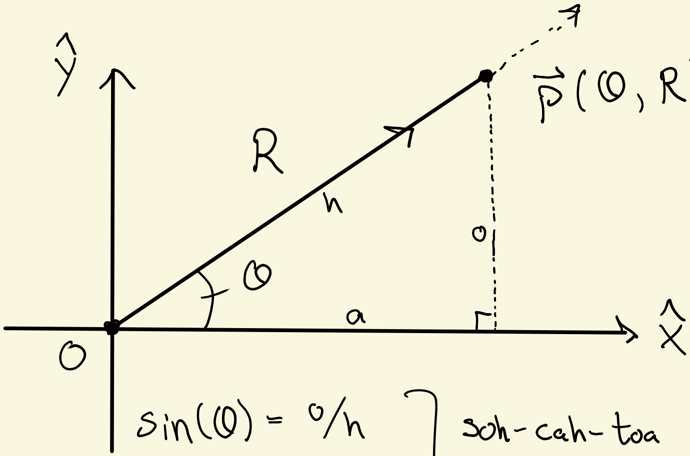
- Trig functions as geometric ratios



$\vec{p}(\theta, R) = (R\cos(\theta), R\sin(\theta))$

$$\boxed{\triangle \quad \begin{array}{l} \sin^{-1}(\theta) := \arcsin(\theta) \\ \sin^{-1}(\theta) \neq 1/\sin(\theta) \end{array}}$$

$$\left.\begin{array}{l} \sin(\theta) = o/h \\ \cos(\theta) = a/h \\ \tan(\theta) = o/a \end{array}\right]$$

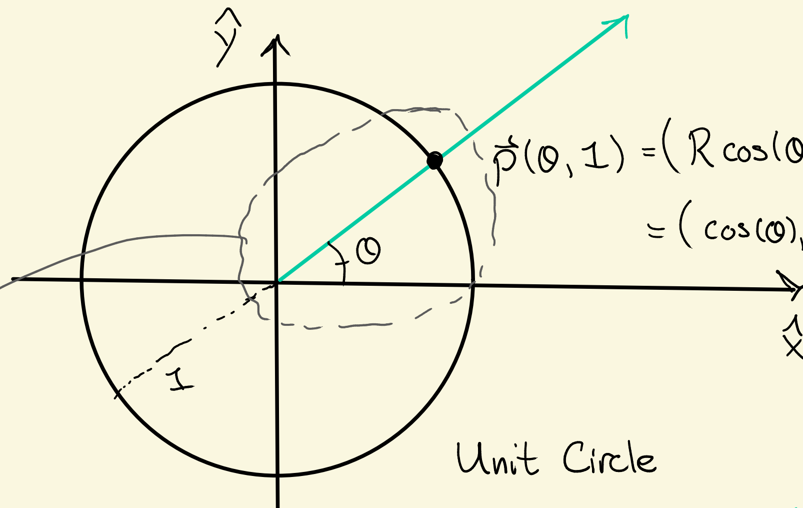
soh-cah-toa

cho-sha-cao

$$\left[\begin{array}{l} \csc(\theta) = 1/\sin(\theta) = h/o \\ \sec(\theta) = 1/\cos(\theta) = h/a \\ \cot(\theta) = 1/\tan(\theta) = a/o \end{array}\right.$$

<u>Problem</u>: Just having θ isn't enough!

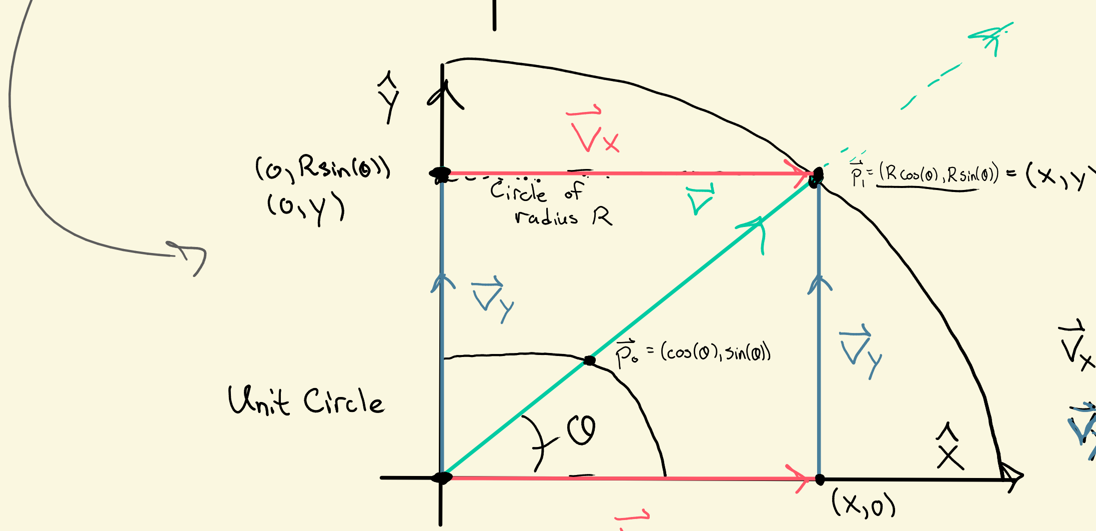
(Given θ, what is $\sin(\theta)$?)



$\vec{p}(\theta, 1) = (R\cos(\theta), R\sin(\theta))$

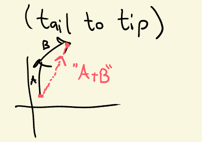
$= (\cos(\theta), \sin(\theta))$

Unit Circle



Recall there is an "addition" for vectors

(tail to tip)



$\vec{V}_x = (R\cos(\theta), 0)$

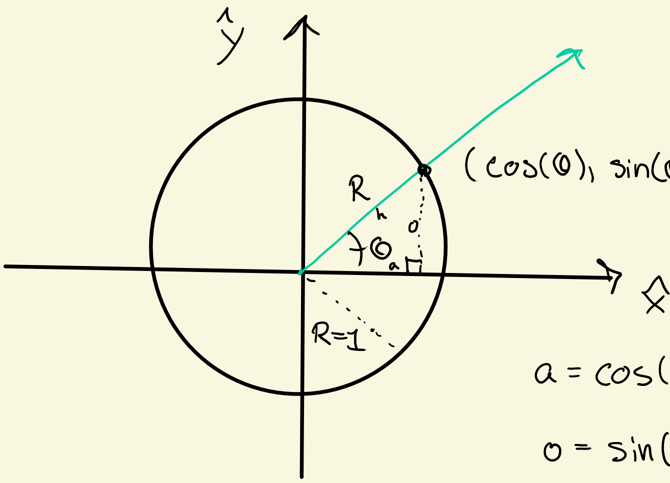
$\vec{V}_y = (0, R\sin(\theta))$

$\boxed{\vec{V} = \vec{V}_x + \vec{V}_y}$

Takeaway: We can break vectors into components (direction)

Slogan: cos $\rightsquigarrow$ $\hat{x}$ component

sin $\rightsquigarrow$ $\hat{y}$ component



$a = \cos(\theta)$

$o = \sin(\theta)$

$h = R = 1$

From Pythagoras, $o^2 + a^2 = h^2$

$\Rightarrow (\cos(\theta))^2 + (\sin(\theta))^2 = 1$

$$\boxed{\Rightarrow \cos^2(\theta) + \sin^2(\theta) = 1.}$$

Fundamental Trig Identity

⚠ Notation!

Try to use $\underbrace{(\cos(\theta))^2}_{\text{use parens}}$ vs $\underbrace{\cos^2\theta + 3}_{\text{not great!}}$

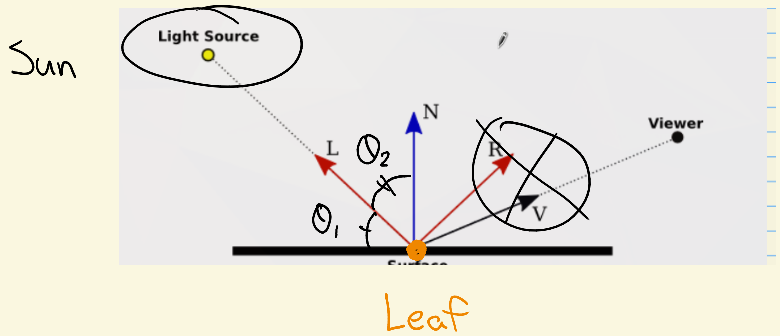
and $\underline{\text{arc}}\sin(\theta)$ vs $\sin^{-1}(\theta)$

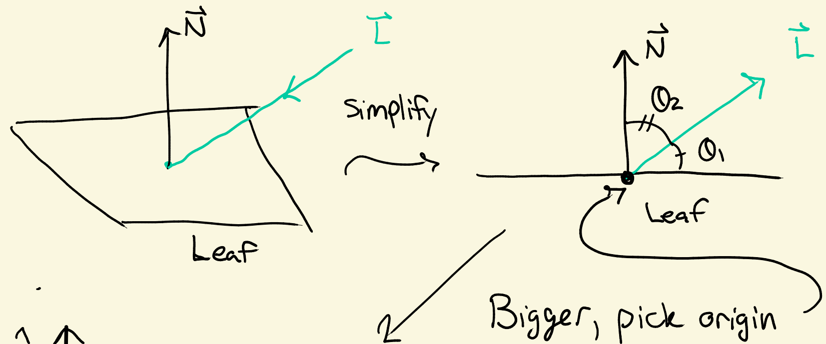
$\underline{\text{arc}}\tan(\theta)$

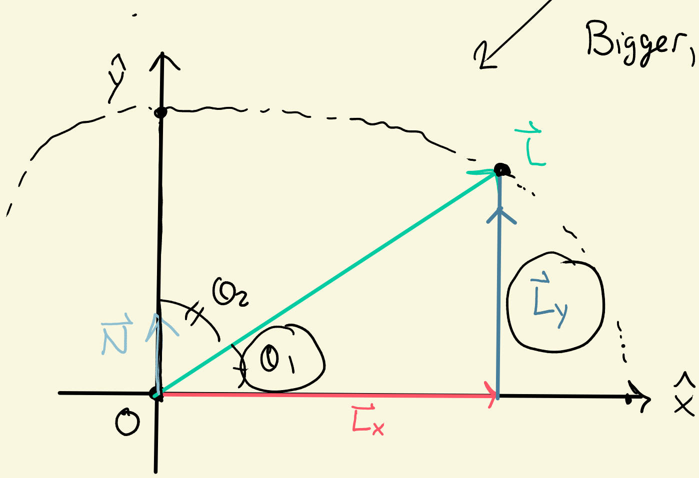
$\underline{\text{arc}}\csc(\theta)$

$\vdots$

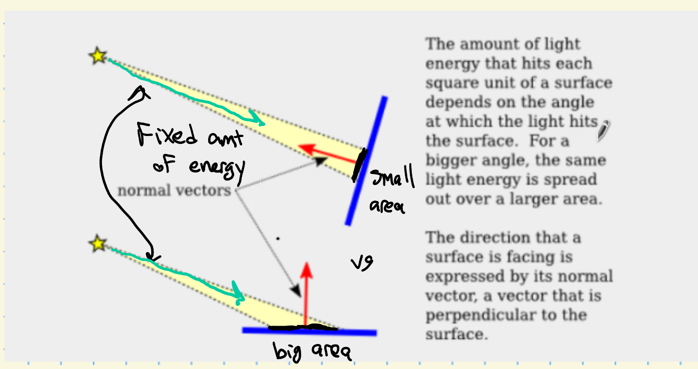
Project 3

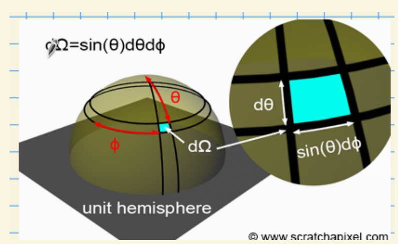






$\underline{E(\vec{L})} \propto \|\vec{L}_y\|$ Only the y component contributes

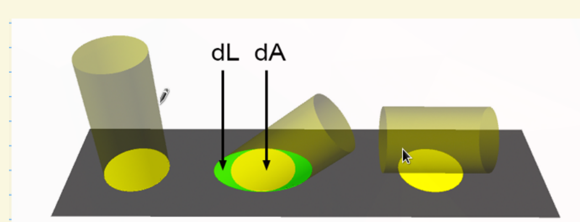




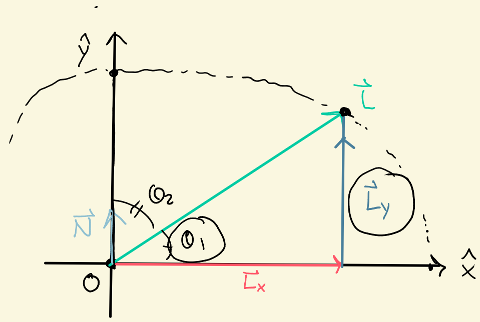
<u>Goal</u>: Find functions for $\|\vec{L}_y\|$

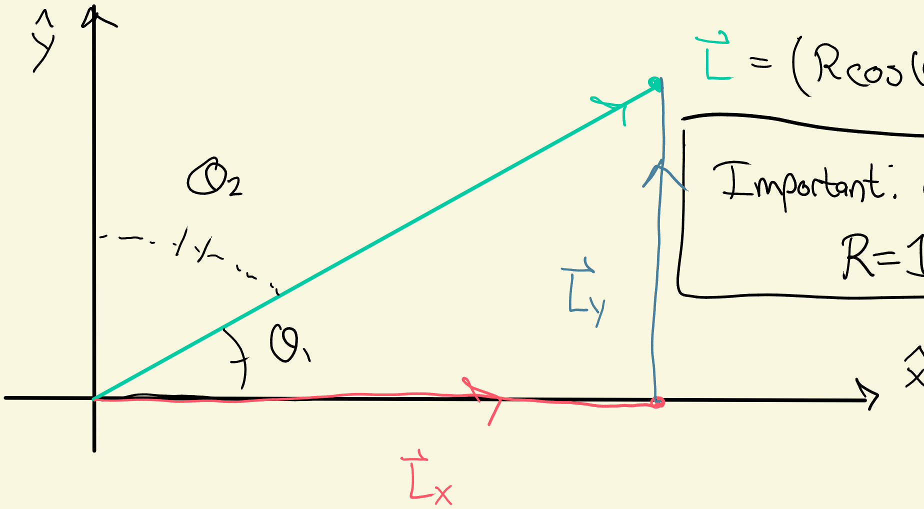
$f(\theta_1) = \ldots$

$g(\theta_2) = \ldots$



Showed For Θ_1:





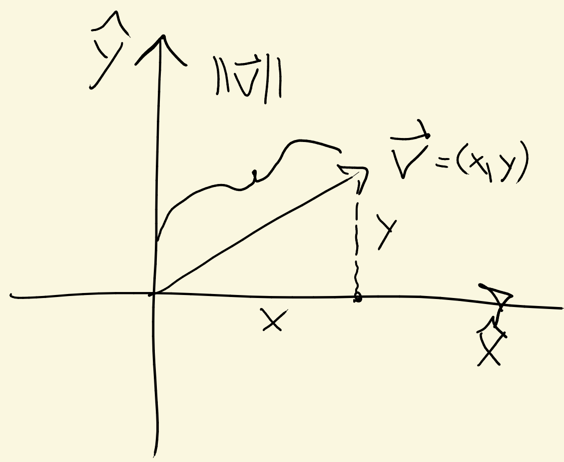
$\vec{L} = (R\cos(\Theta_1), R\sin(\Theta_1))$

Important: can assume $R=1$

$\vec{L} = (\cos(\Theta_1), \sin(\Theta_1))$

$\vec{L}_x = (\cos(\Theta_1), 0)$

$\boxed{\vec{L}_y = (0, \sin(\Theta_1))}$



$\vec{V} = (x,y)$ then $\|\vec{V}\| = \sqrt{x^2+y^2}$

$\|\vec{L}_y\| = \sqrt{0^2 + (\sin(\Theta_1))^2}$

$= \sin(\Theta_1)$

Conclusion: $E(\vec{L}) \propto \sin(\Theta_1)$