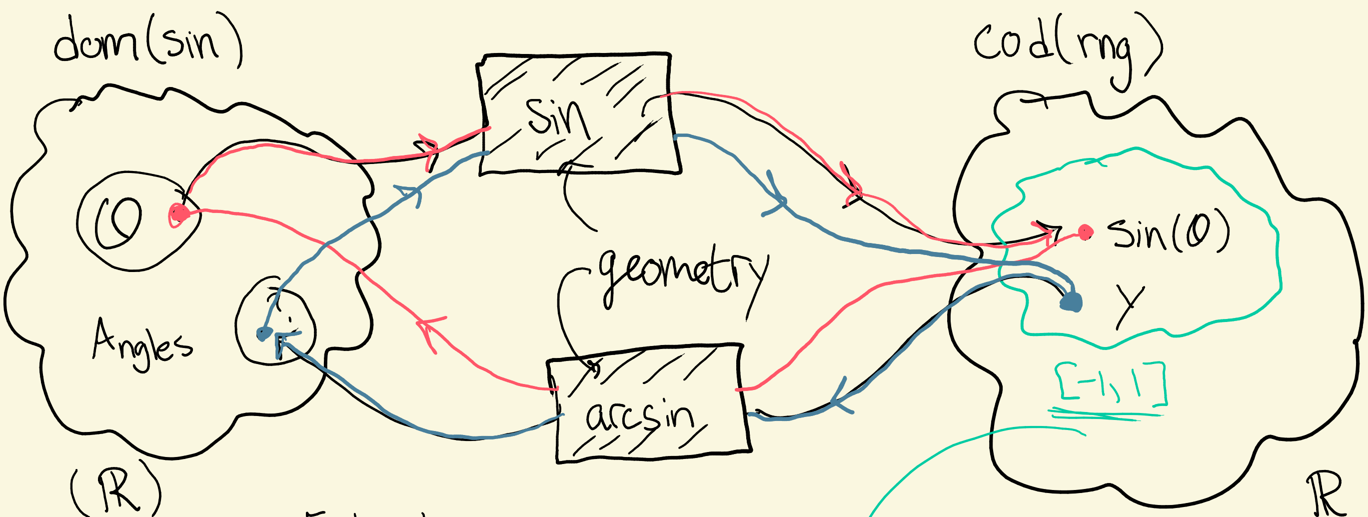
Tuesday Dec 1^{st}

- Schedule
  - Today: 4.5B
  - Thurs: 5.1
  - Tues*: 5.2
- Course Evaluations

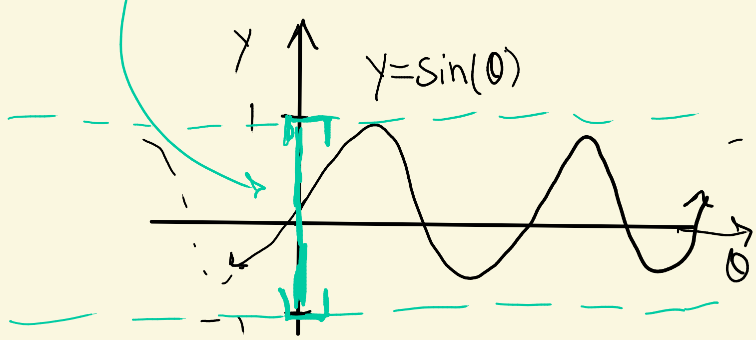
4.7: Inverse trigonometric functions



We'd like

Extract

- $\arcsin(\sin(\theta)) = id(\theta) = \theta$ ①
- $\sin(\arcsin(y)) = id(y) = y$



$\underline{\text{Ex}}$ Compute $\underline{\sin^{-1}(3/5)}$

$\left(\underline{\arcsin}(3/5) \right)$

$\uparrow y$

$\left(\triangle \neq \dfrac{1}{\underbrace{\sin(3/5)}} \right)$

Have a name: $\csc(3/5)$

$\underline{\underline{\theta}} = \arcsin(3/5)$

$*$ $\leftarrow$ injectivity

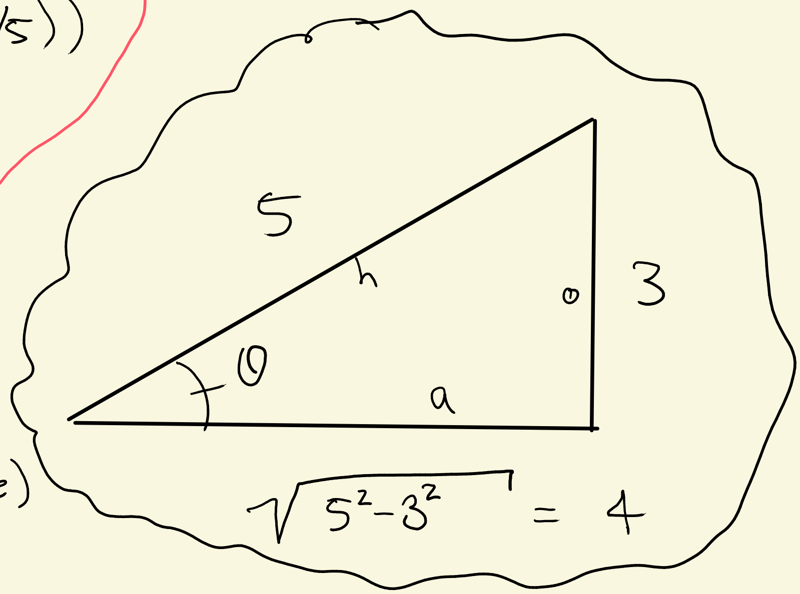
$\Rightarrow \sin(\theta) = \underline{\sin(\arcsin}(3/5))$

$* \Rightarrow \boxed{\sin(\theta) = 3/5}$

Functional inverse

Out of luck!

(Not a special angle/triangle)



$o^2 + a^2 = h^2 \Rightarrow a = \sqrt{h^2 - o^2}$

$\underline{\text{But!}}$ We $\underline{\text{can}}$ compute things like

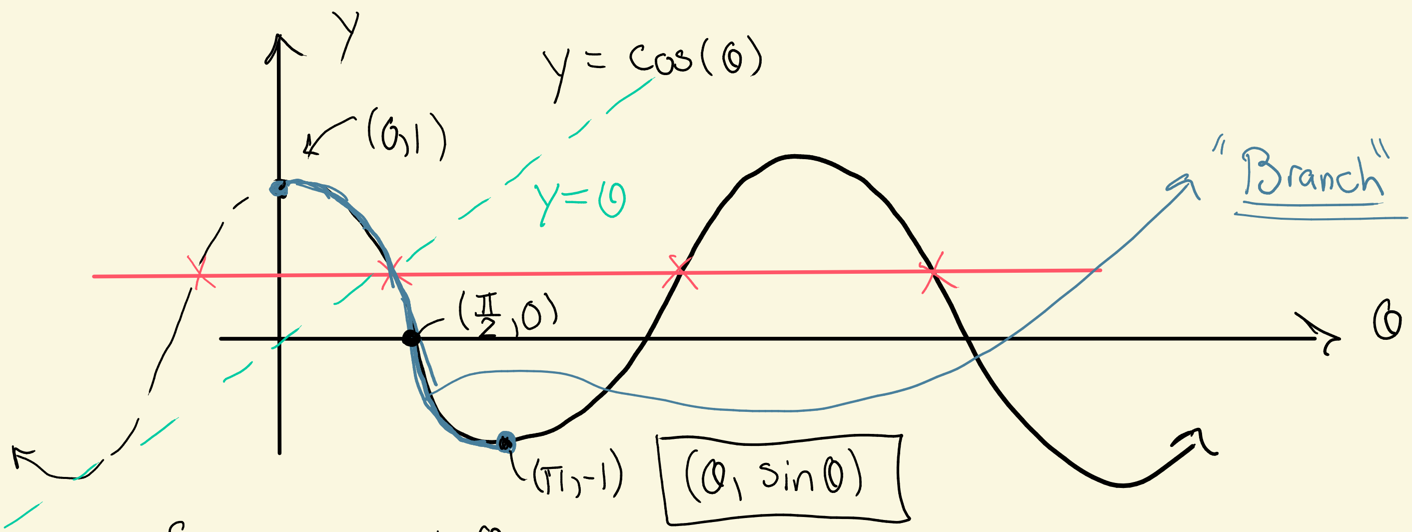
- $\cos(\theta)$
- $\tan(\theta)$
- $\sec(\theta)$

$\underline{\text{Ex}}$: $\boxed{\cos(\arcsin(3/5))} = \cos(\theta) = \underline{\underline{4/5}}.$

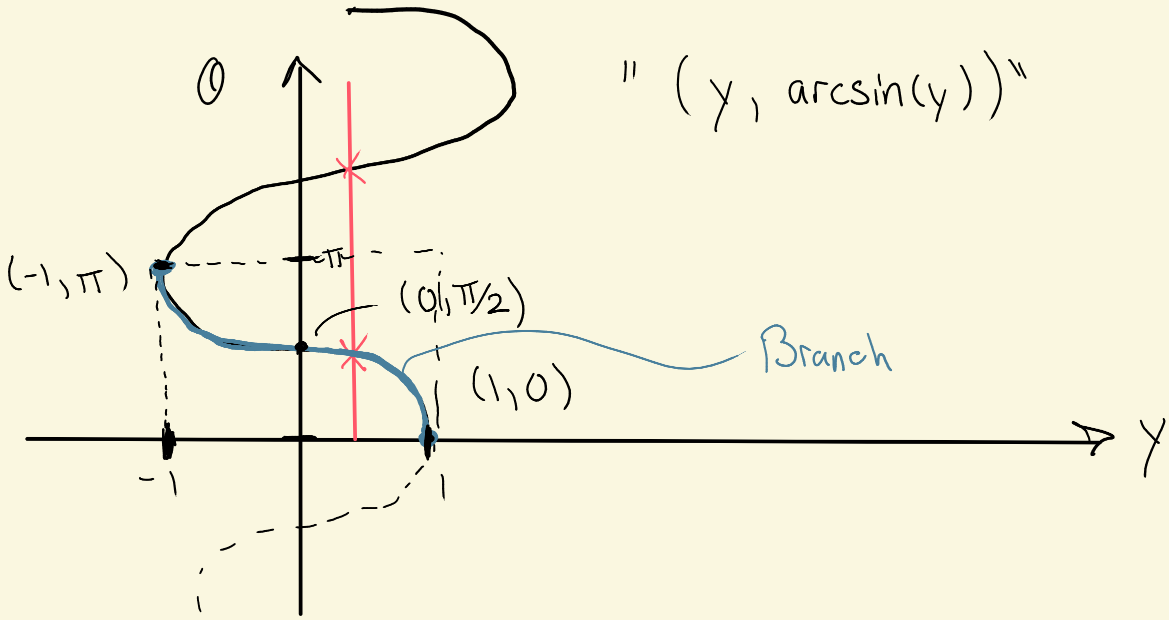
Number!

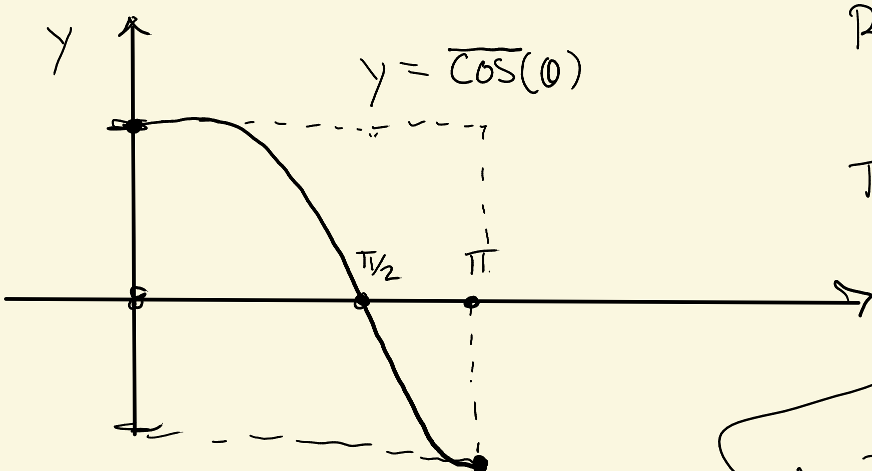
Comes up in integral Calculus!

Consider cos, arccos



Swap y and θ





Restrict domain of cos to $[0,\pi]$

Then $\to$ $\overline{\cos}$ is injective

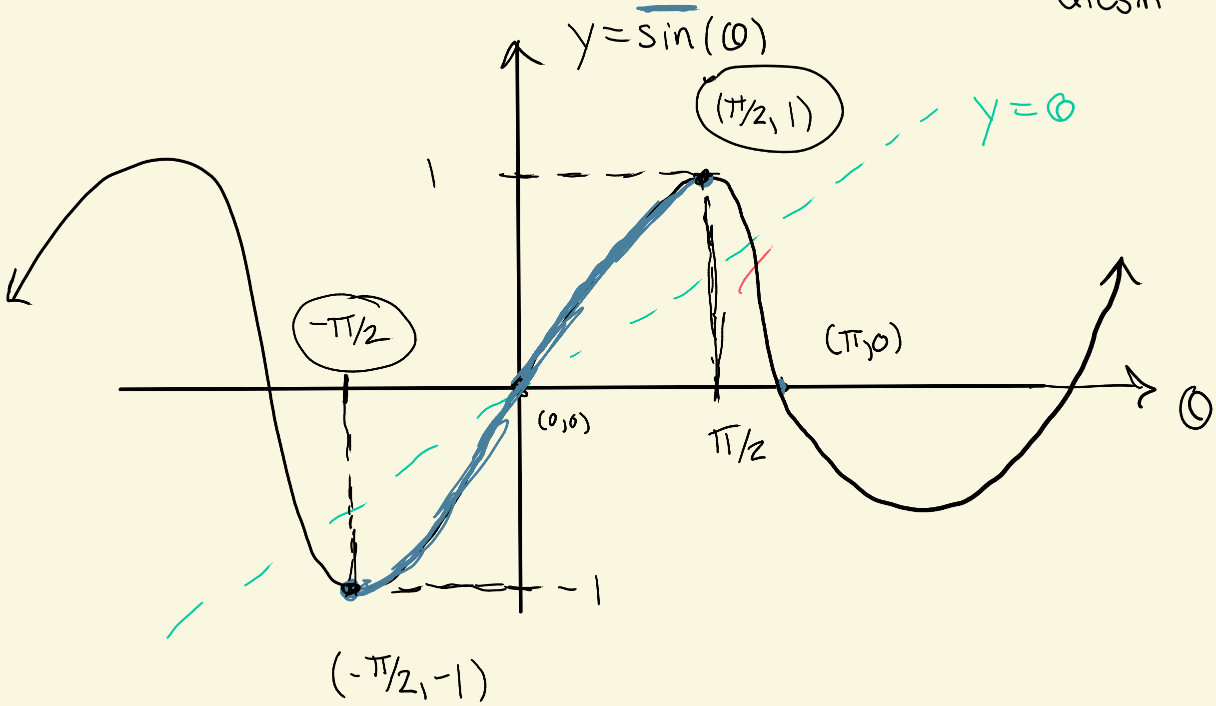
$\Rightarrow$ arccos is a function.

$\text{rng}(\cos) = [-1,1]$

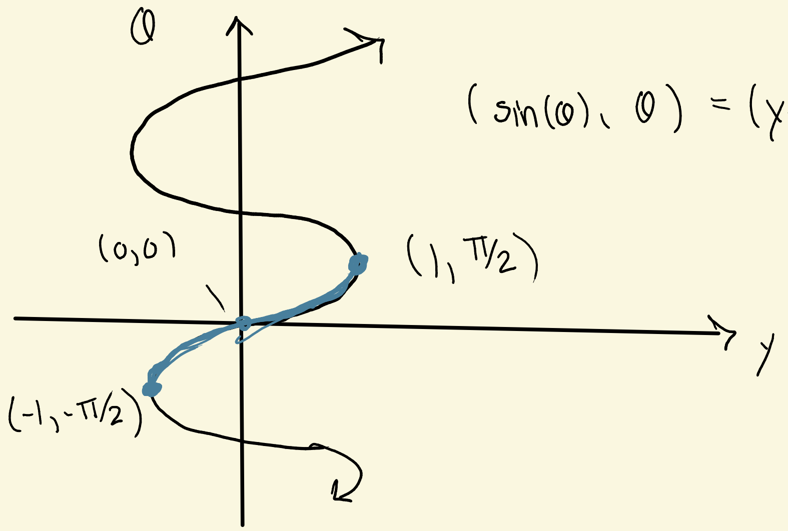
$$\boxed{\begin{aligned} \Rightarrow \ \text{dom}(\arccos) &= \text{rng}(\overline{\cos}) = [-1,1] \\ \text{rng}(\arccos) &= \text{dom}(\overline{\cos}) = [0,\pi] \end{aligned}}$$

Similar analysis for $\sin(\theta)$

Define arcsin



$(\sin(\theta), \theta) = (y, \arcsin(y))$



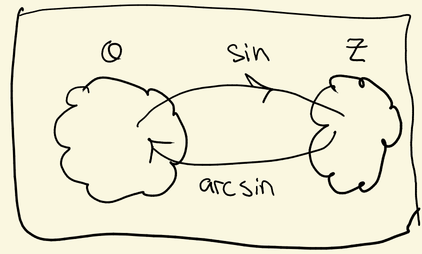
?

$\Rightarrow$ $\text{dom}(\arcsin) = \text{rng}(\sin) = [-1, 1]$ ✓

$\text{rng}(\arcsin) = \text{dom}(\sin) = \boxed{[-\pi/2, \pi/2]}$

Upshot:

- $\sin(\arcsin(z)) = z$ if $z \in [-1,1]$
- $\arcsin(\sin(\theta)) = \theta$ if $\boxed{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]}$ (1)
- $\cos(\arccos(z)) = z$ if $z \in [-1,1]$
- $\boxed{\arccos(\cos(\theta)) = \theta}$ if $\boxed{\theta \in [0, \pi]}$ (2)



Computation

$\vdots$

$\Rightarrow \cos(\theta) = ?$

ok b/c injectivity

$\Rightarrow \boxed{\arccos(\cos(\theta))} = \arccos(?)$

$\Rightarrow \boxed{\theta} = \arccos(?)$ Need to check!