Thursday Dec 3rd

(Fix worksheets)

5.1 (?) : Simplifying trig fns

Goal: Reduce complicated $\to$ simpler

Strategy: Writing everything in terms of cos / sin

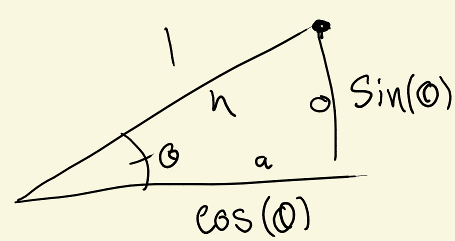
Eg Simplify $F = \underbrace{\left(\dfrac{\sin(\theta)\tan(\theta)}{\cot(\theta)}\right)\cos(\theta)\csc(\theta)}$

Boxing it up

Let $s = \sin(\theta)$
$c = \cos(\theta)$

sin	son	csc	cho
cos	cah	sec	sha
tan	toa	cot	ca o

$x \mapsto \frac{1}{x}$



$$\Rightarrow F = \left(\frac{s\,(s/c)}{(c/s)}\right) c \cdot (1/s)$$

$$= s \cdot (s/c) \cdot (c/s)^{-1} \cdot c \cdot (s)^{-1}$$

$$= s \cdot (s/c) \cdot (s/c) \cdot (c/s)$$

$$= s \cdot (s/c)$$

unbox

$$\Rightarrow F = \sin(\theta) \cdot \left(\frac{\sin(\theta)}{\cos(\theta)}\right) \Rightarrow$$

$$= \sin(\theta) \left(\frac{\cos(\theta)}{\sin(\theta)}\right)^{-1}$$

$$\boxed{F = \begin{cases} \boxed{\sin^2(\theta)/\cos(\theta)}, \\ \text{or} \\ \bullet\ \sin(\theta)\cdot\tan(\theta), \\ \text{or} \\ \bullet\ \sin(\theta)/\cot(\theta). \end{cases}}$$

- Verifying identities

$$\boxed{F(\theta) = g(\theta)}$$

If you work and end up at

- $1 = 1$
- $0 = 0$

$$\cdots$$
$$\Rightarrow x = y$$
$$\Rightarrow \cos(x) = \cos(y)$$
$$\vdots$$
$$1 = 1$$

Method

1) Start with $f(\theta)$

$\overbrace{\cdots\cdots}^{\text{steps}} \longrightarrow$ end up with $g(\theta)$

$$
\begin{aligned}
f(\theta) &= \ ? \\
\Rightarrow \quad &= \ ? \\
&= \ ? \\
\Rightarrow \quad & \ \vdots \\
\Rightarrow \quad &= g(\theta)
\end{aligned}
$$

2) Same except backward: start w/ $g(\theta) \rightsquigarrow F(\theta)$

$$
\begin{aligned}
g(\theta) &= \ ? \\
&= \ ? \\
&= \ ? \\
& \ \vdots \\
&= F(\theta)
\end{aligned}
$$

3) Show $f(\theta) = h(\theta)$ and $g(\theta) = h(\theta)$

$$\begin{aligned} f(\theta) &= \; ? \\ &= \; ? \\ &\;\vdots \\ &= h(\theta) \end{aligned} \qquad \text{and} \qquad \begin{aligned} g(\theta) &= \; ? \\ &= \; ? \\ &= \; ? \\ &\;\vdots \\ &= h(\theta) \end{aligned}$$

$$\begin{tikzcd} f(\theta) \arrow[rr, leftrightarrow, "="] \arrow[dr] & & g(\theta) \arrow[dl] \arrow[ll, bend right] \\ & h(\theta) \arrow[ul, dashed, bend left] \arrow[ur, dashed, bend right] & \end{tikzcd}$$

Eg Show

$$\underbrace{\sin(-\theta) + \csc(\theta)}_{f(\theta)} = \boxed{\underbrace{\cot(\theta)\cos(\theta)}_{g(\theta)}}$$

$$\Rightarrow \quad g(\theta) = \cot(\theta)\cos(\theta)$$

$$= \left(\frac{\cos(\theta)}{\sin(\theta)}\right)\cos(\theta)$$

$$= \frac{\cos^2(\theta)}{\sin(\theta)}$$

$$= \frac{1 - \sin^2(\theta)}{\sin(\theta)}$$

Aside

$$s^2 + c^2 = 1$$

$$\Rightarrow c^2 = 1 - s^2$$

$$= \frac{1 - \sin^2(\theta)}{\sin(\theta)}$$

$$= \frac{1}{\sin(\theta)} - \frac{\sin^2(\theta)}{\sin(\theta)}$$

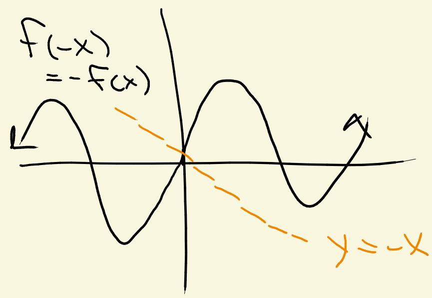
$g(\theta) = h(\theta)$

$$\boxed{= \csc(\theta) - \sin(\theta) = \boxed{h(\theta)}}.$$

$$\boxed{f(\theta)} = \sin(-\theta) + \csc(\theta)$$

sin is odd

$$= \csc(\theta) + \sin(-\theta)$$



$\sin(-\theta) = -\sin(\theta)$

$$= \csc(\theta) - \sin(\theta)$$

$$\boxed{= h(\theta)}.$$

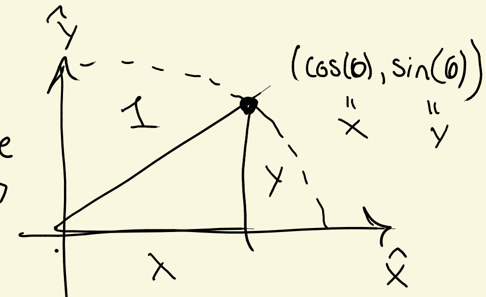
Done!

Important Trig Identities

- Pythagoras for polar coords:

$$\boxed{\sin^2(\theta) + \cos^2(\theta) = 1}$$

Memorize



$$x^2 + y^2 = 1^2$$

$/\sin^2(\theta)$ • $1 + \left(\frac{\cos(\theta)}{\sin(\theta)}\right)^2 = \left(\frac{1}{\sin(\theta)}\right)^2$

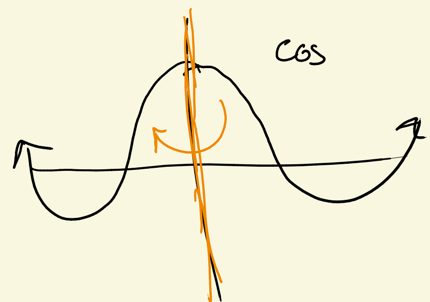
$$\boxed{\Rightarrow 1 + \cot^2(\theta) = \csc^2(\theta)}$$

Don't memorize!

$/\cos^2(\theta)$ • $\left(\frac{\sin(\theta)}{\cos(\theta)}\right)^2 + 1 = \left(\frac{1}{\cos(\theta)}\right)^2$

$$\boxed{\Rightarrow \tan^2(\theta) + 1 = \sec^2(\theta)}$$

- Even/odd



$\cos(\theta)$ is <u>even</u>: $\cos(-\theta) = +\cos(\theta)$

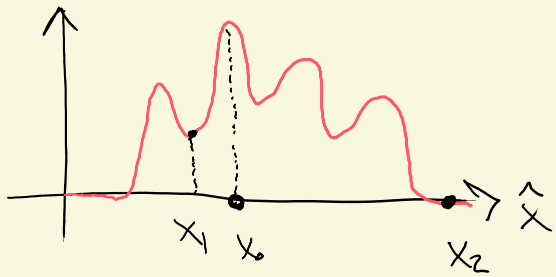
$\sin(\theta)$ is <u>odd</u>: $\sin(-\theta) = -\sin(\theta)$

- Double/half angle identities

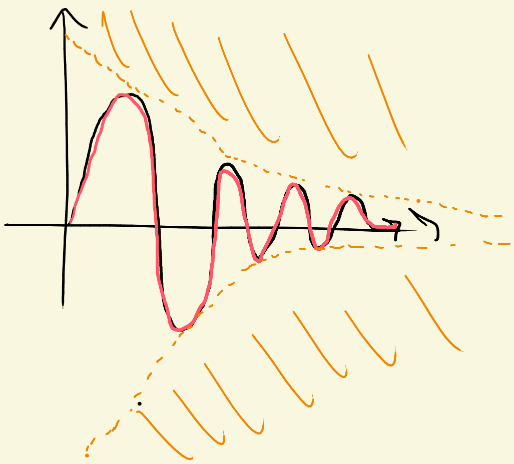
- $\underbrace{\sin(\theta + \psi)}_{\text{"Friendly" :)}} = \sin(\theta)\cos(\psi) + \cos(\theta)\sin(\psi)$

- $\underbrace{\cos(\theta + \psi)}_{\text{cliquey! :(}} = \underbrace{\cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)}_{\text{"Superposition"}}$

one single wave + phase shift



General combination of waves



"Damped" wave

- $\underline{\sin(2\theta)} = \sin(\theta+\theta)$

$$\sin(\theta+\theta) = \sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta)$$
$$= \underline{2\sin(\theta)\cos(\theta)}$$

- $\underline{\cos(2\theta)} = \cos(\theta+\theta)$

$$\cos(\theta+\theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$$
$$= \cos^2(\theta) - \sin^2(\theta)$$

$\boxed{\text{⚠ Not equal to 1!}}$

$= \cos^2(\theta) - \sin^2(\theta)$

$s^2 + c^2 = 1$

$= \cos^2(\theta) - (1 - \cos^2(\theta))$

$= 2\cos^2(\theta) - 1$

Play a similar game to get

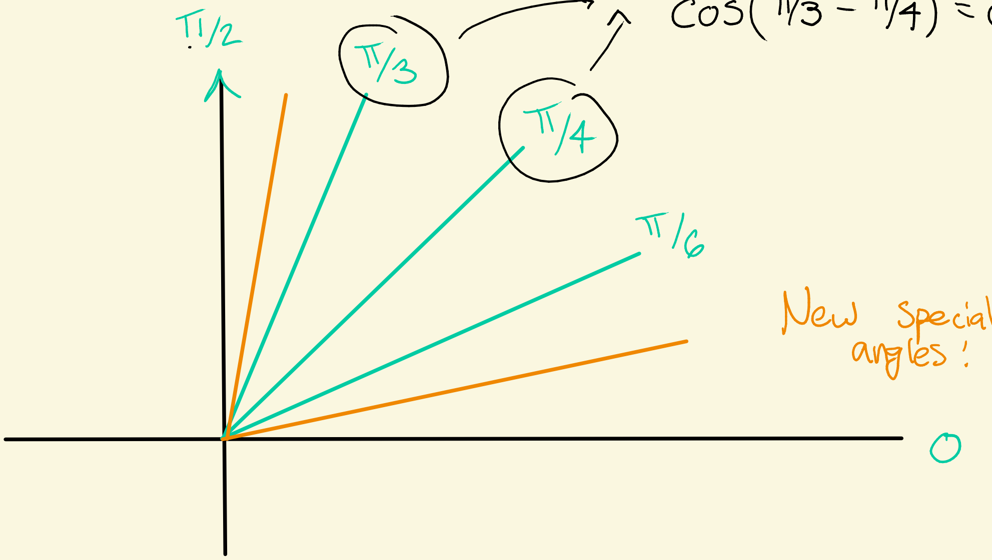
$\cos(2\theta) = 2\cos^2(\theta) - 1$

- $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$

- $\underbrace{\sin(\theta+\psi)}_{\text{"Friendly"}} = \sin(\theta)\cos(\psi) + \cos(\theta)\sin(\psi)$
- $\underbrace{\cos(\theta+\psi)}_{\text{Clever!}} = \underbrace{\cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)}$

Motivation

$\cos(\pi/3 - \pi/4) = \cos(\pi/12)$



$3 \cdot 4 + 4 = 16$ special angles

$+\ 4$ new ones