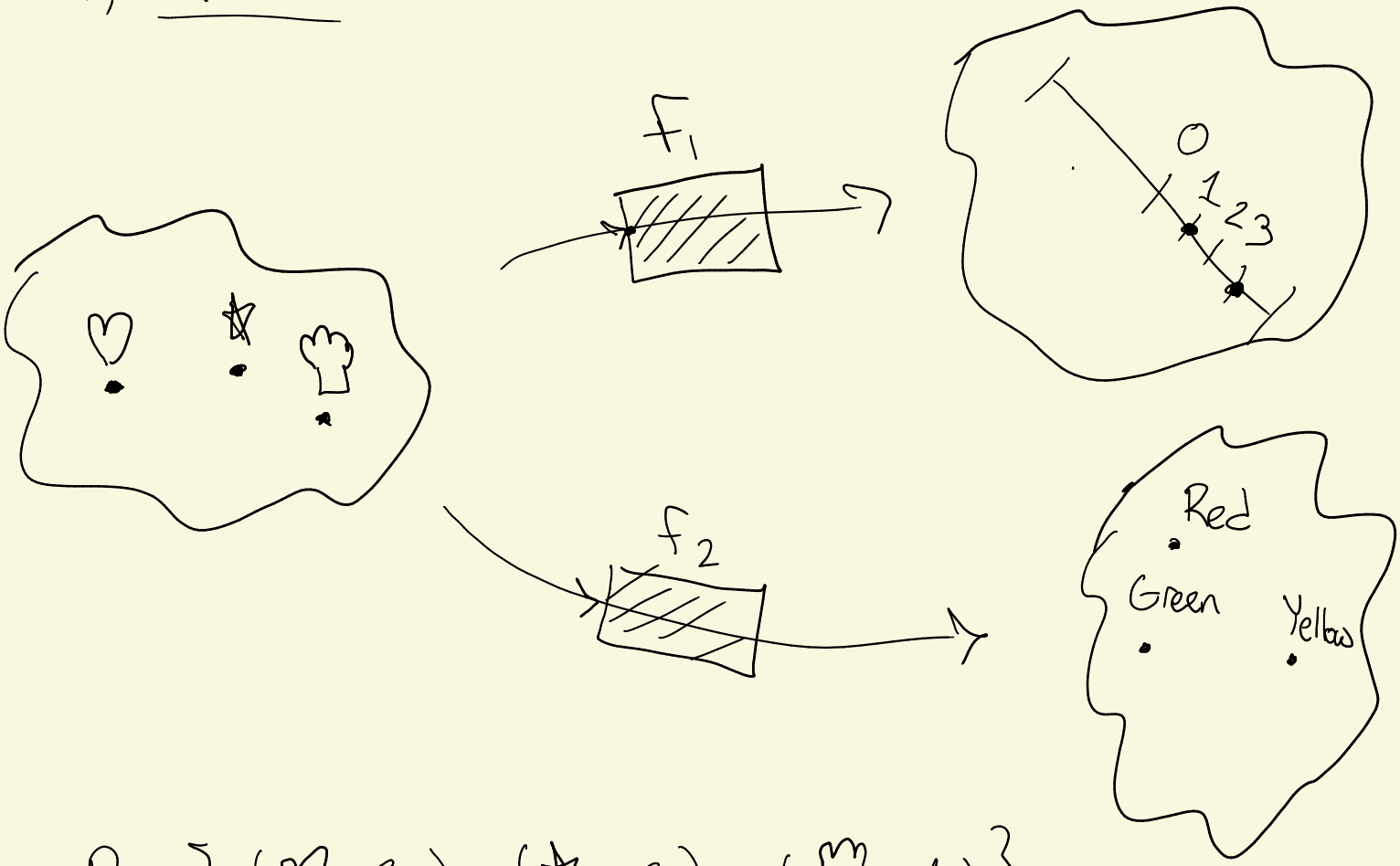
Quiz #2 Review

Sections: 1.3, 1.4, 1.5 (up until Tuesday)

- Domains & Ranges

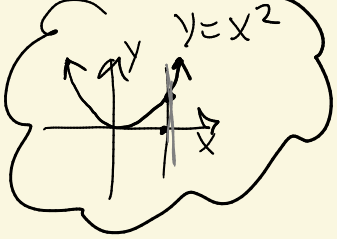
1) Abstract

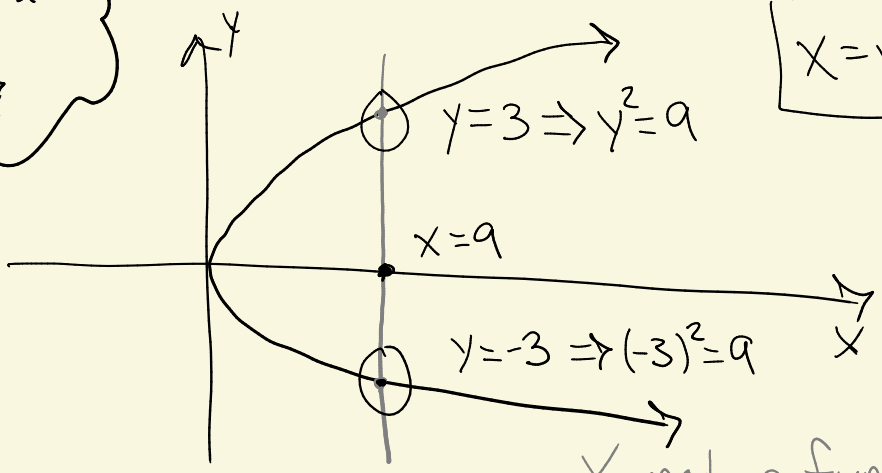


$R_1 = \{(♡, 3), (☆, 3), (👕, 1)\}$

$Range(R_1) = \{3, 3, 1\} = \{3, 1\}$

$R_2 = \{(♡, \text{"Red"}), (☆, \text{"Yellow"}), (👕, \text{"Green"}),$
$(♡, \text{"Green"})\}$ ← not a function



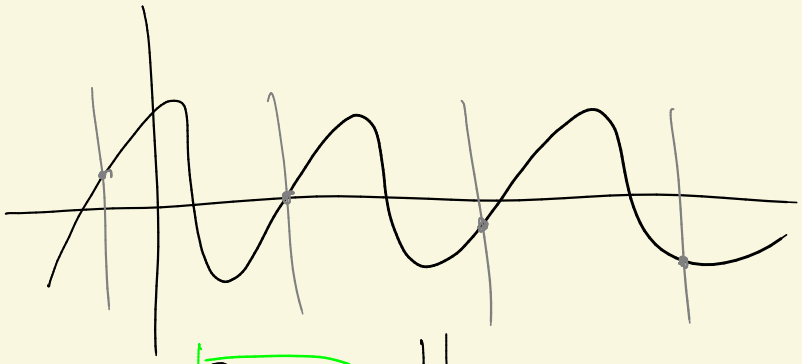


$x=y^2$

$(9,3)$

$(9,-3)$

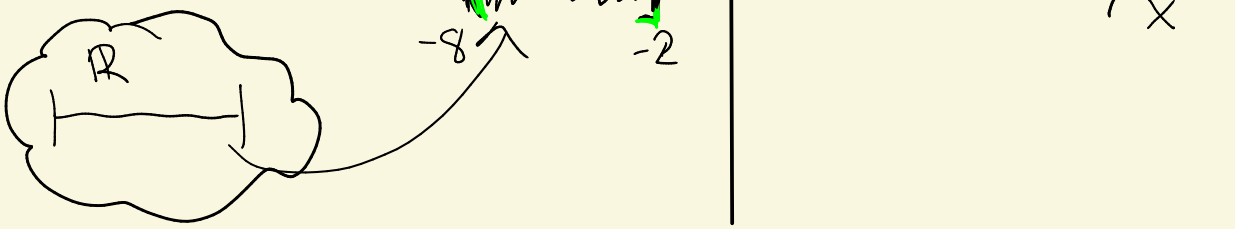
✗ not a function



✓ function

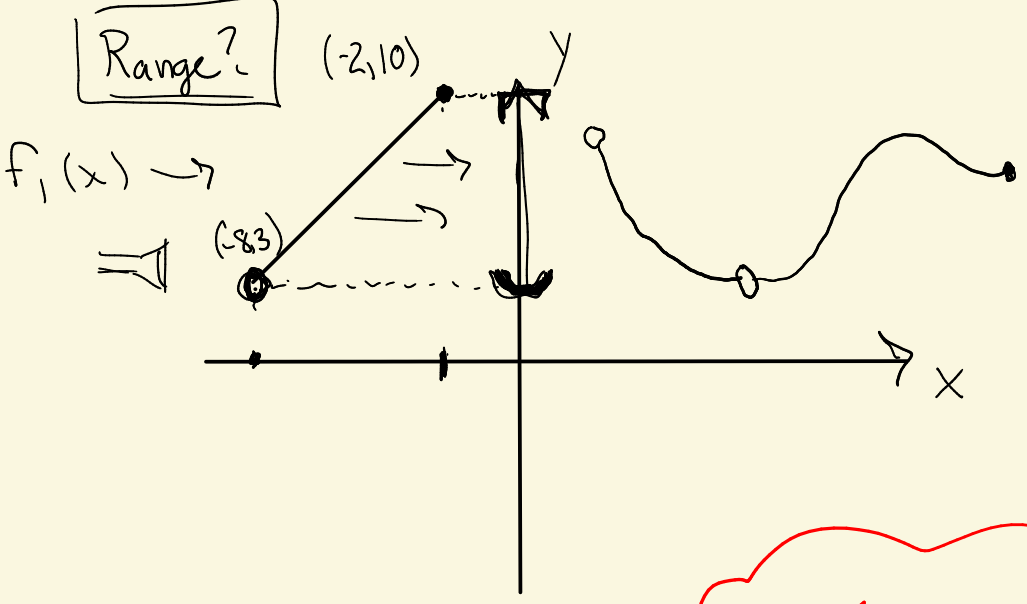
Domain

$f_1(x)$



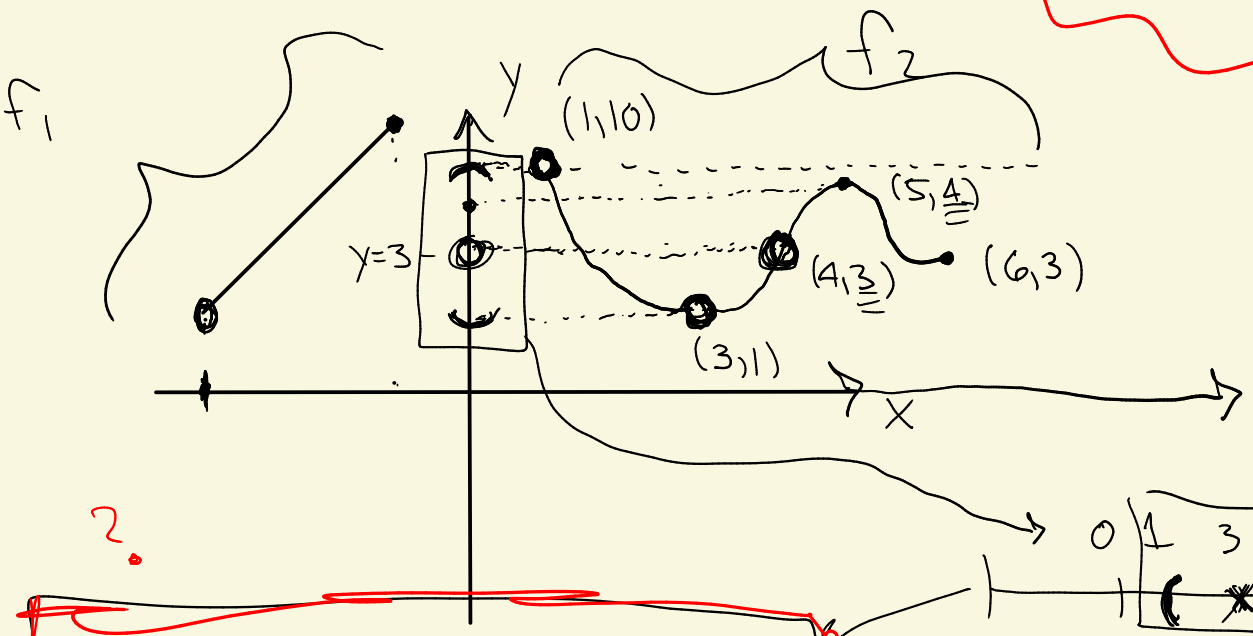
$dom(f_1) = (-8,-2]$

Range?



$\text{Range}(f_1) = (3, 10]$

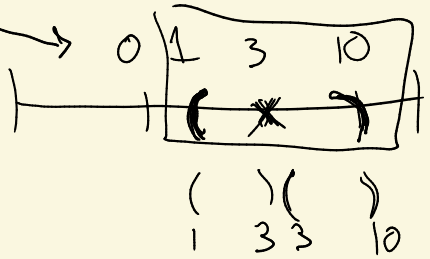
⚠ Warning: Common Mistake!



2.

$\text{Range}(f_2) = (1, 10) \setminus \{3\}$

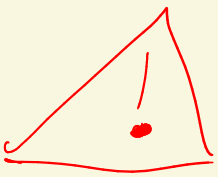
$= (1,3) \cup (3,10)$

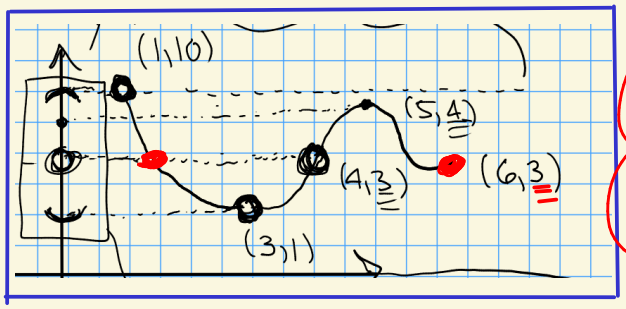


Not actually correct!

What went wrong?

There are other x values with $f_2(x) = 3$!





So 3 is in the range, and $\text{rng}(f) = (1, 10]$.

- Domains and Ranges from formulas

Eg: $f(x) = \dfrac{3-x}{\sqrt{x^2-1}}$

Default hope: $\left.\begin{aligned} \text{dom}(f) &= \mathbb{R} \\ \text{rng}(f) &= \mathbb{R} \end{aligned}\right\}$ All real numbers

- Question: where does $f(x)$ have a "problem"?

Eg 1

$\sqrt{(?)}$

$? = x$

$? = x-3$

$? = x^2+4x+2$

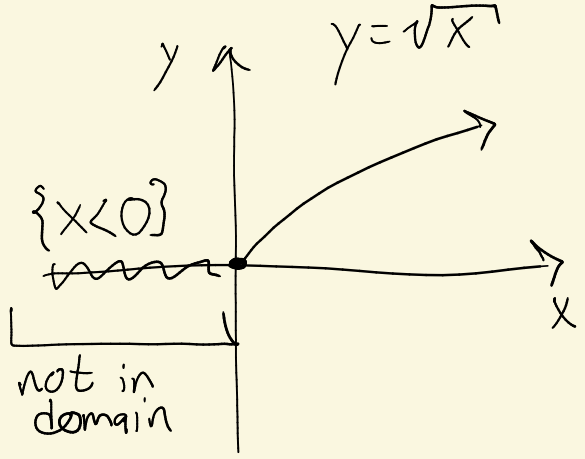
$\sqrt{x} \rightsquigarrow \sqrt{(?)}$

- "Problems" when $(?) < 0$
- "Okay" when $(?) \geq 0$

- $f(x) = \sqrt{x}$

$\text{dom}(f) = \mathbb{R} \setminus \{x \in \mathbb{R} \mid x < 0\}$

$= [0, \infty)$



Eg. 2

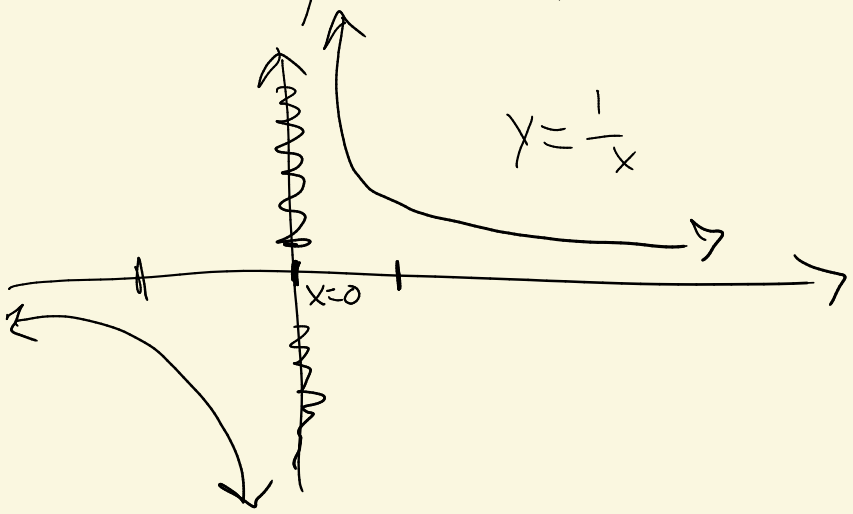
$\frac{1}{(?)}$

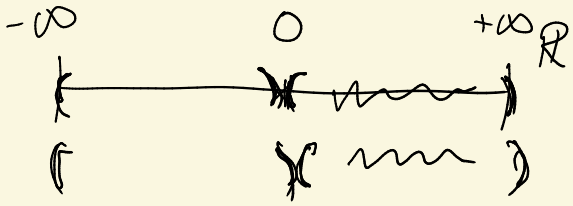
- "Problems" when $(?) = 0$
- "Okay" when $(?) \neq 0$

$?= x$

$?= x-3$

$?= x^2+4x+2$





$\text{dom}(1/x) = \mathbb{R} \setminus \{0\}$

$= (-\infty, 0) \cup (0, \infty)$

Eg $f(x) = \frac{10}{\sqrt{x-\pi}}$

"$\sqrt{(?)}$"

"$1/(?)$"

1) $\sqrt{(?)}$ Problems $(?) < 0$

$(?) = x-\pi \rightsquigarrow x-\pi < 0$

$\Rightarrow x < \pi$

Throw out $(-\infty, \pi)$

2) $1/(?)$ Problems $(?) = 0$

$(?) = \sqrt{x-\pi} \rightsquigarrow \sqrt{x-\pi} = 0 \Rightarrow x = \pi$

Throw out $\{\pi\}$

$$\mathrm{dom}(f) = \left(\mathbb{R} \setminus \{ x \in \mathbb{R} \mid x < \pi \}\right) \setminus \{\pi\}$$

$$= (\pi, \infty).$$