Section 1.4

1. (1 point) Determine a *point-slope* equation for a line through $(-5,2)$ and $(6,-3)$.

Formula: $y - y_0 = m(x - x_0)$ $\left\{ \begin{array}{l} m = \text{slope} \\ (x_0, y_0) = \text{point.} \end{array} \right.$

Here $(x_0, y_0) = (-5, 2)$ (We could pick either point)

and $m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$

so take $(x_1, y_1) = (6, -3)$, then

$$m = \frac{(-3) - 2}{6 - (-5)} = \frac{-5}{6+5} = \frac{-5}{11},$$

So our formula is

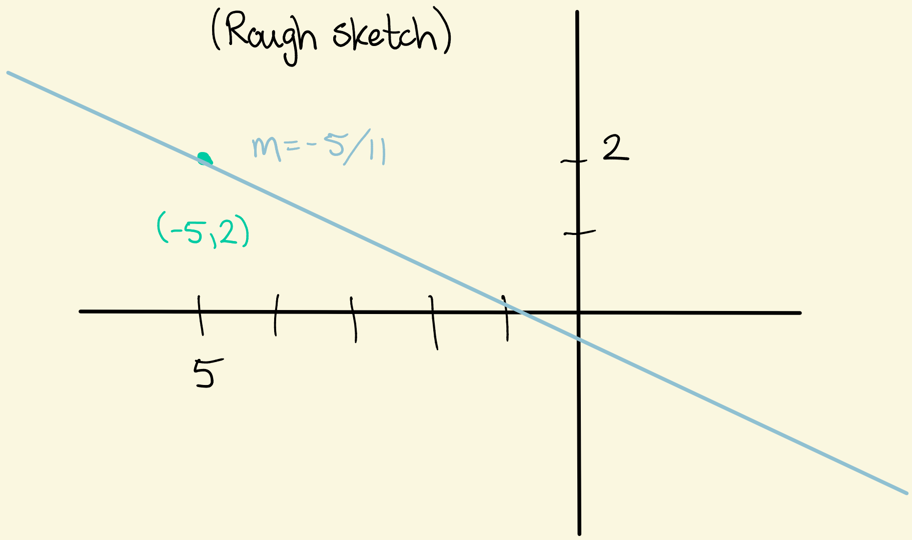
$$y - 2 = \left(\frac{-5}{11}\right)(x - (-5))$$

$$\Rightarrow \boxed{y - 2 = \left(\frac{-5}{11}\right)(x + 5).}$$

What's the picture?

(Could check intercepts and plot exactly.)

(Rough sketch)



2. (2 points) Write the *slope-intercept* equation for a line through the point $(2, 3)$ that is perpendicular to the line $2x + 4y - 7 = 0$

We need a theorem:

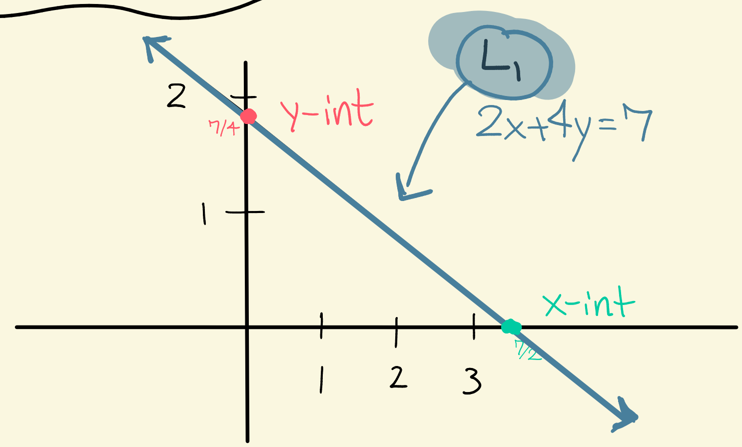
If L_1 is a line with slope m, then any line L_2 that is perpendicular to L_1 has slope $-1/m$.

What's the picture?

Our equation: $2x+4y=7$

x-int: $y=0 \Rightarrow 2x=7 \Rightarrow x=7/2$, so it's $(7/2, 0)$

y-int: $x=0 \Rightarrow 4y=7 \Rightarrow y=7/4$, so it's $(0, 7/4)$



Slope of L_1:

$$2x+4y-7=0 \Rightarrow 4y=-2x+7$$
$$\Rightarrow y = (-1/2)x + (7/4)$$
$$\Rightarrow m = -1/2$$

The perpendicular line L_2 thus has slope $-\frac{1}{m} = \frac{-1}{(-1/2)} = -(2/-1) = +2.$

We now have $\left\{ \begin{array}{ll} \text{A slope} & 2 \\ \text{A point} & (2,3) \end{array} \right\} \Rightarrow$ Can use point-slope formula.

Let $(x_0, y_0) = (2,3)$ and $m=2$, then the line L_2 we want is given by

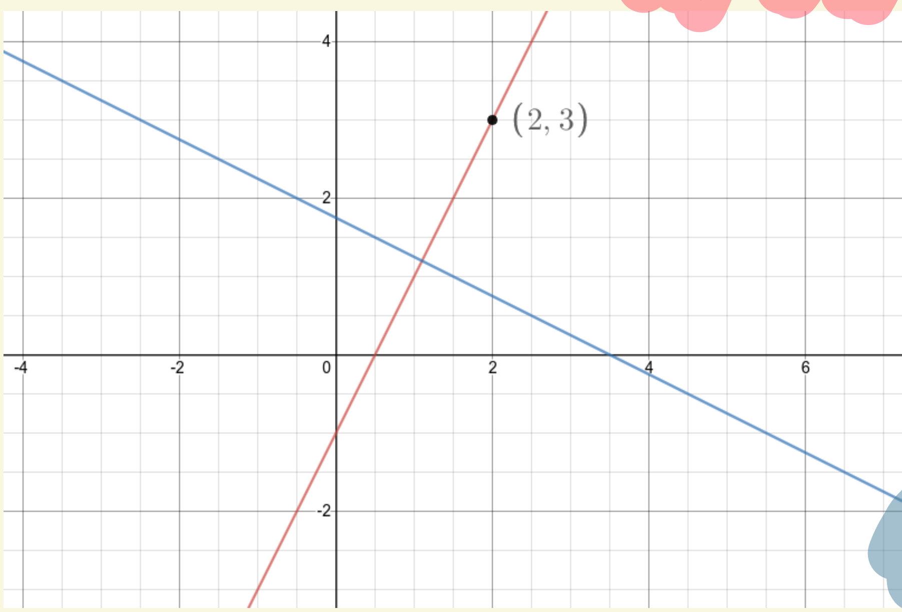
$$y - y_0 = m(x - x_0) \Rightarrow y-3 = 2(x-2)$$
$$\Rightarrow y = (2x-4)+3$$

$$\boxed{\Rightarrow y = 2x - 1.}$$

What's the picture?

L_2: $y = 2x - 1$

Slope $m = 2$



L_1: $2x + 4y = 7$

Slope $m = -1/2$

3. (2 points) Given the function defined $f(x) = x^2 + 3$, determine the average rate of change of $f(x)$ from $x_1 = 2$ to $x_2 = 4$.

We need a fact:

$$\boxed{\text{Avg. Rate of Change}\big((x_1, f(x_1)), (x_2, f(x_2))\big) := \text{slope of the line between } (x_1, f(x_1)) \text{ and } (x_2, f(x_2)) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.}$$

Here $\begin{cases} \cdot\ x_1 = 2 \Rightarrow f(x_1) = (2)^2 + 3 = 7 \\ \cdot\ x_2 = 4 \Rightarrow f(x_2) = (4)^2 + 3 = 19 \end{cases}$ $\xrightarrow{\text{yields points}}$ $p_1 = (2, 7)$, $p_2 = (4, 19)$

$\Rightarrow$ Avg. Rate of change $(p_1, p_2) = \dfrac{19 - 7}{4 - 2} = \dfrac{12}{2} = \boxed{6.}$

What's the picture?