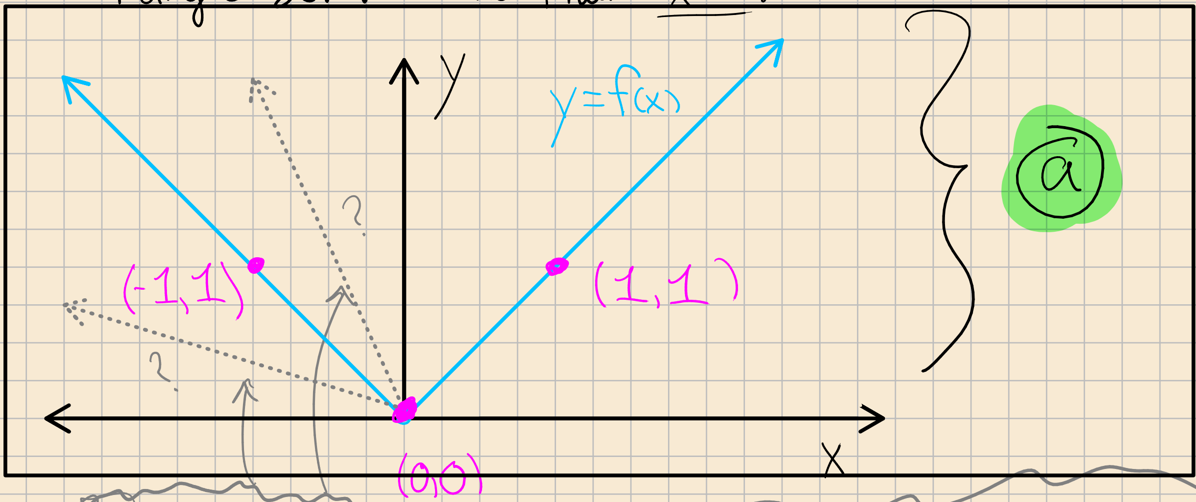
1.6.a Solns

1. (3 points) For this problem, let $f(x) = |x|$.

(a) Graph $f(x) = |x|$. Then determine the domain and range of $f(x)$.

(b) Graph and label $f(x+2)$ on the coordinate system above. Then determine the domain and range of $f(x+2)$.

What are some "interesting" points? Think about where the function changes behavior: here, near $x=0$.



Need 3 points, because 2 points only determine a single line! Without a 3rd point, we don't know which of these possibilities happens.

Domain and Range

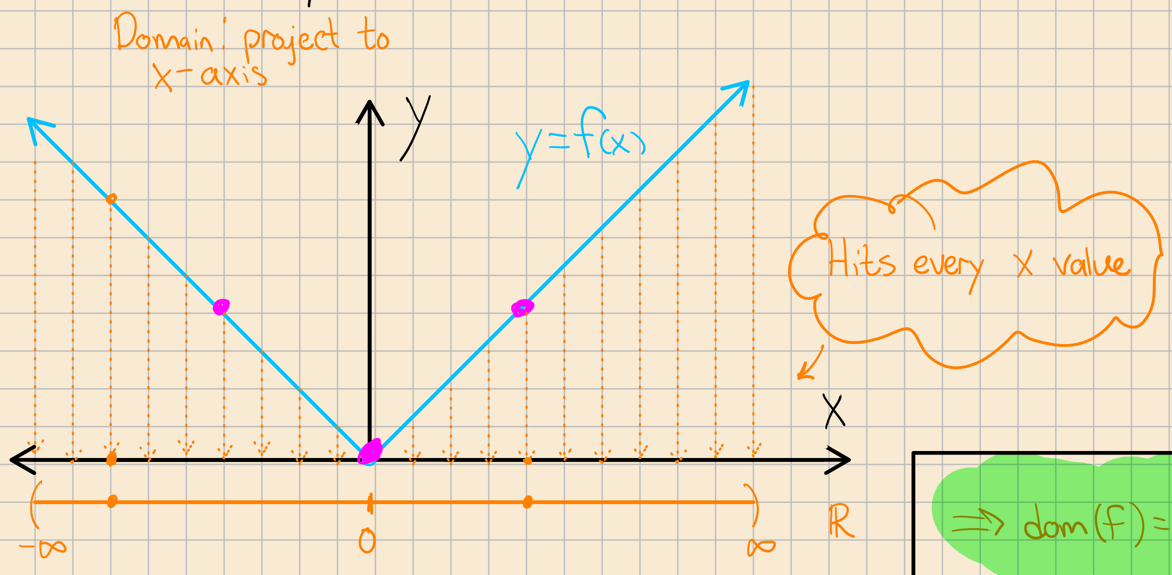
Hope: both are all of $\mathbb{R} = (-\infty, \infty)$. What are the problematic points? None! For all $x \in (-\infty, \infty)$, $f(x) = |x|$ makes sense.

$\boxed{\Rightarrow \text{domain}(f) = \mathbb{R} = (-\infty, \infty)}$

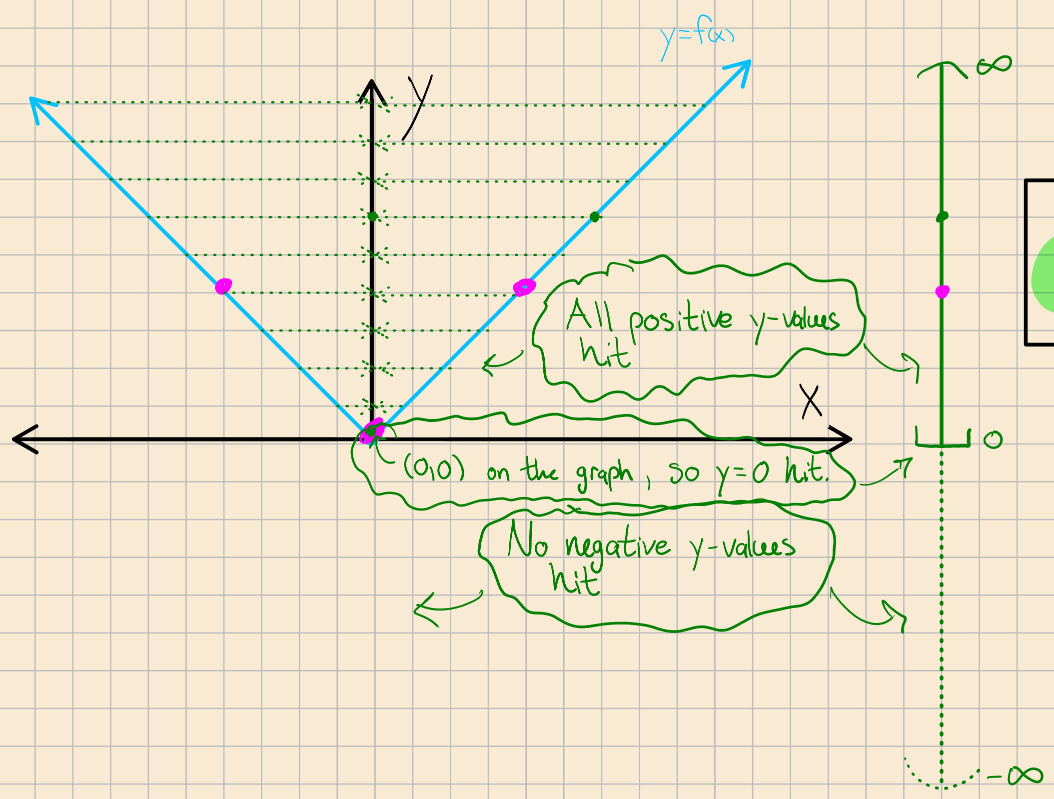
Range: use the fact that $|x| \geq 0$ for all $x \in \text{dom}(f)$, so

$\boxed{\text{range}(f) = [0, \infty).}$

Alternatively, the projection method:



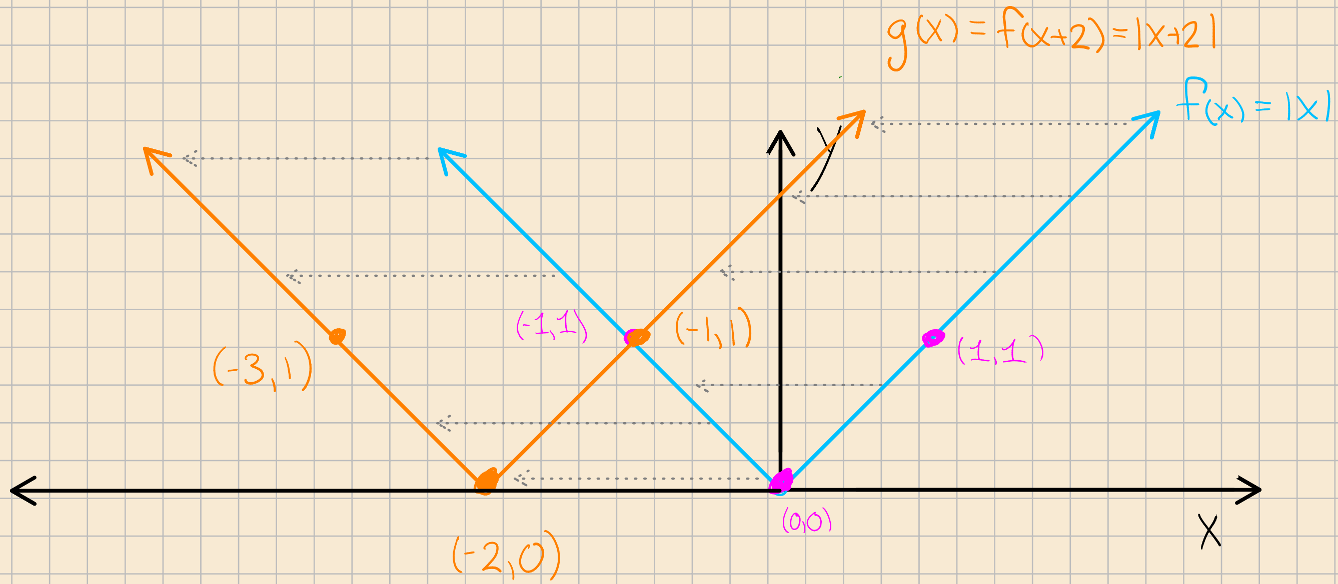
$\Rightarrow \text{dom}(f) = \mathbb{R}.$



$\Rightarrow \text{range}(f) = [0, \infty).$

(b) Tricky: Shifts inside a function have "opposite" effects

- $f(x)+2$: Shifts graph up by 2
- $f(x+2)$: Shifts graph left by 2



But in this instance, note that this doesn't change the x or y projections.

$$\boxed{\Rightarrow \text{domain}(g) = \mathbb{R} = (-\infty,\infty), \quad \text{range}(g) = [0,\infty).}$$

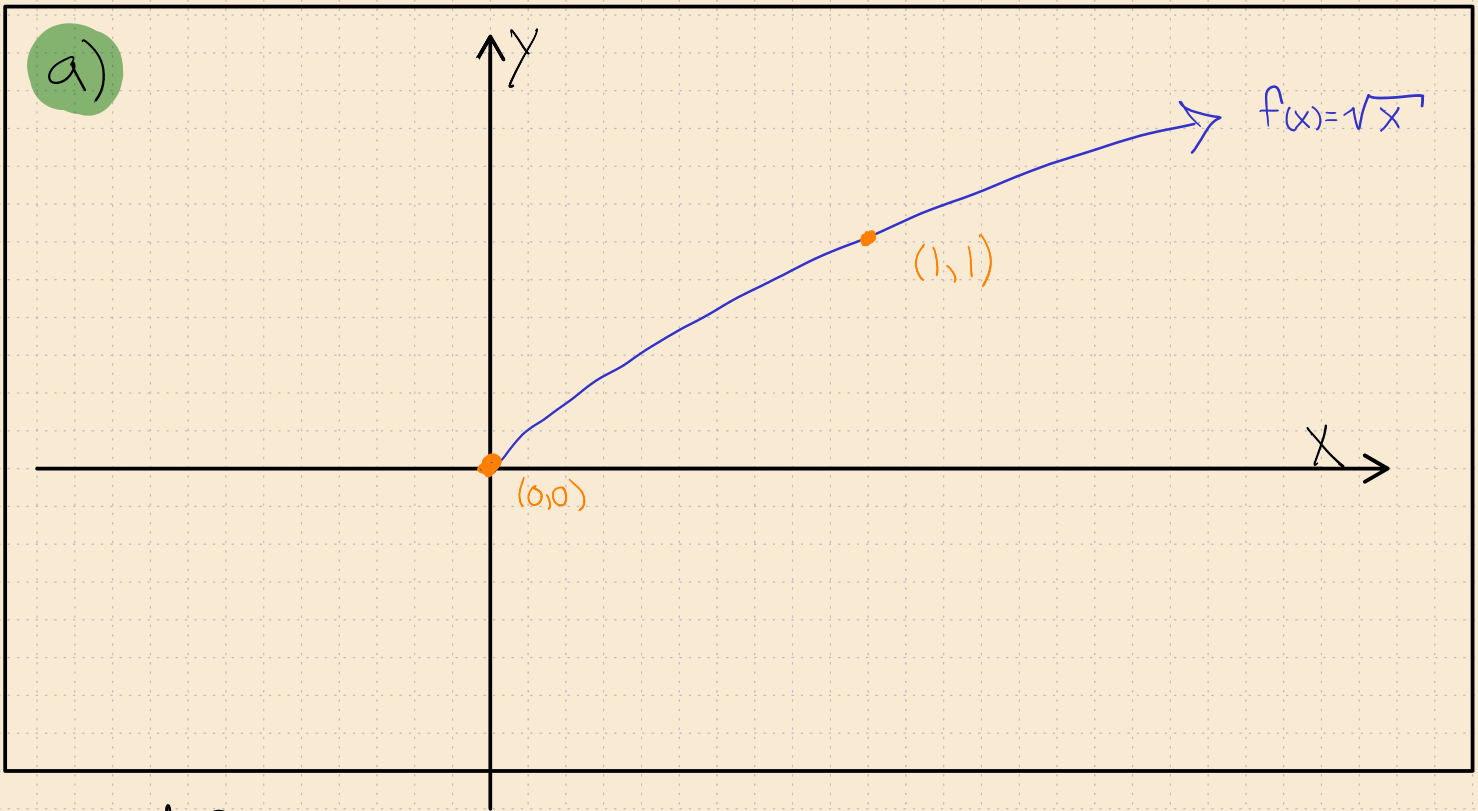
2. (3 points) For this problem, let $f(x) = \sqrt{x}$.

(a) Graph $f(x) = \sqrt{x}$. Then determine the domain and range of $f(x)$.

(b) Graph and label $f(x) - 3$ on the coordinate system above. Then determine the domain and range of $f(x) - 3$.

· What are "interesting" points? The function f changes behavior near $x=0$, namely it is not defined as soon as $x<0$.

a)



Domain & Range:

Domain: Hope it's all of $\mathbb{R}$, but what x-values are problematic?

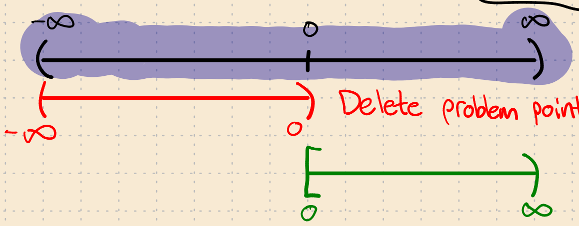
Answer: any $x<0$ (note $x=0$ is okay.)

Hope: all of $\mathbb{R}$

So $\text{domain}(f) = \mathbb{R} \setminus \{x \in \mathbb{R} \mid x<0\}$

$= (-\infty,\infty) \setminus (-\infty,0)$

$= [0,\infty)$.

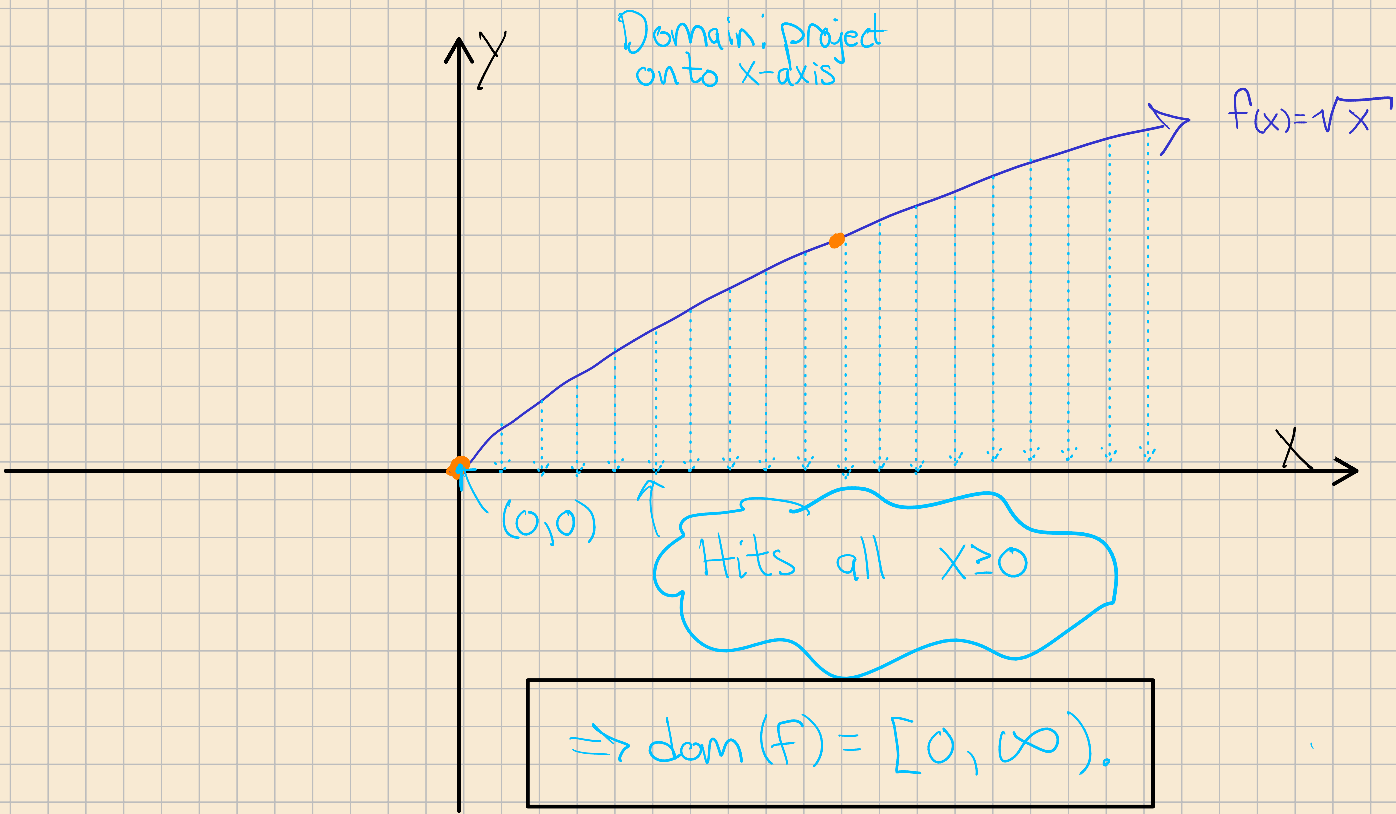


Set of good points remaining = domain

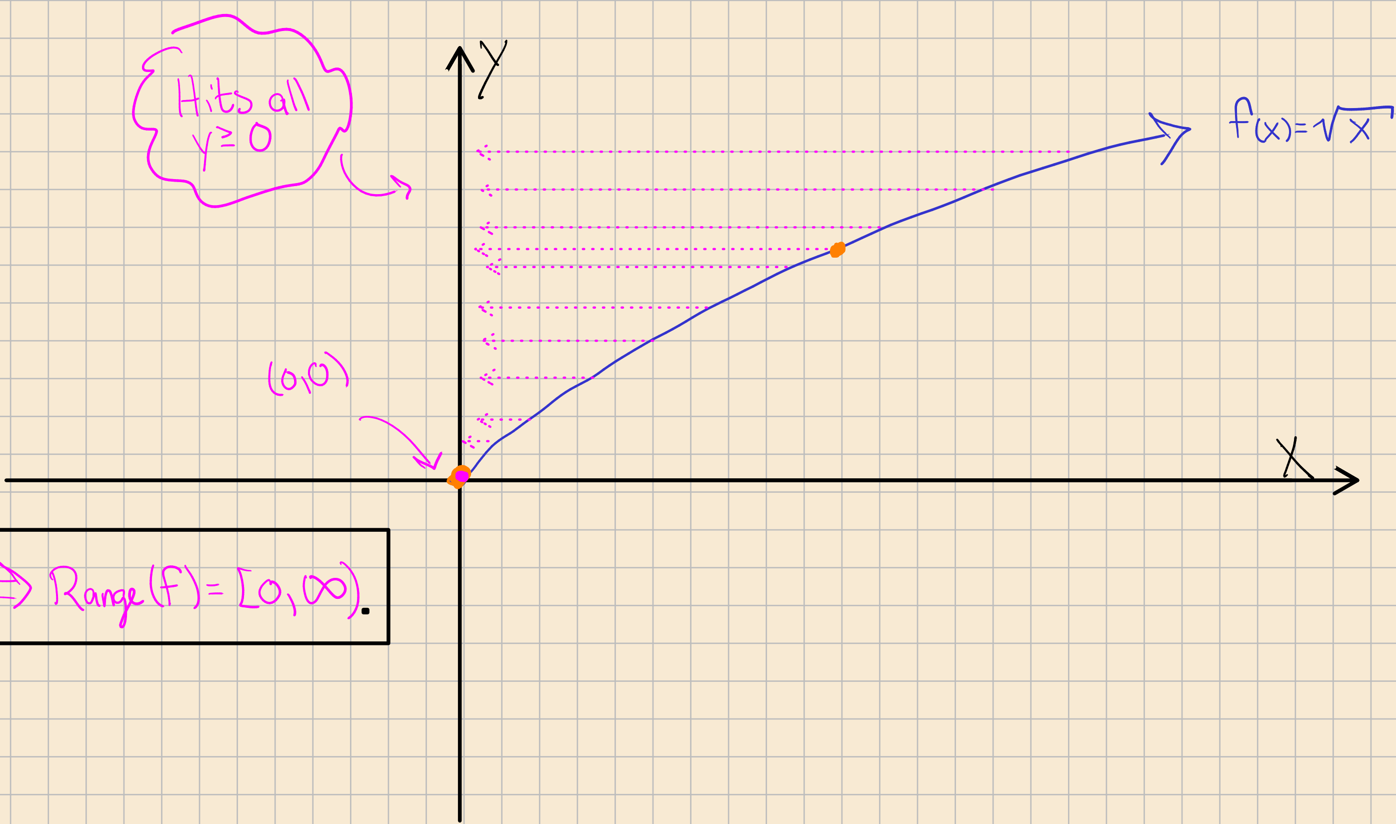
Range: Use the fact that $\sqrt{x} \geq 0$ for all x in $\text{domain}(\sqrt{x})$

$\Rightarrow \text{range}(f) = [0,\infty)$.

Alternatively, we can project:

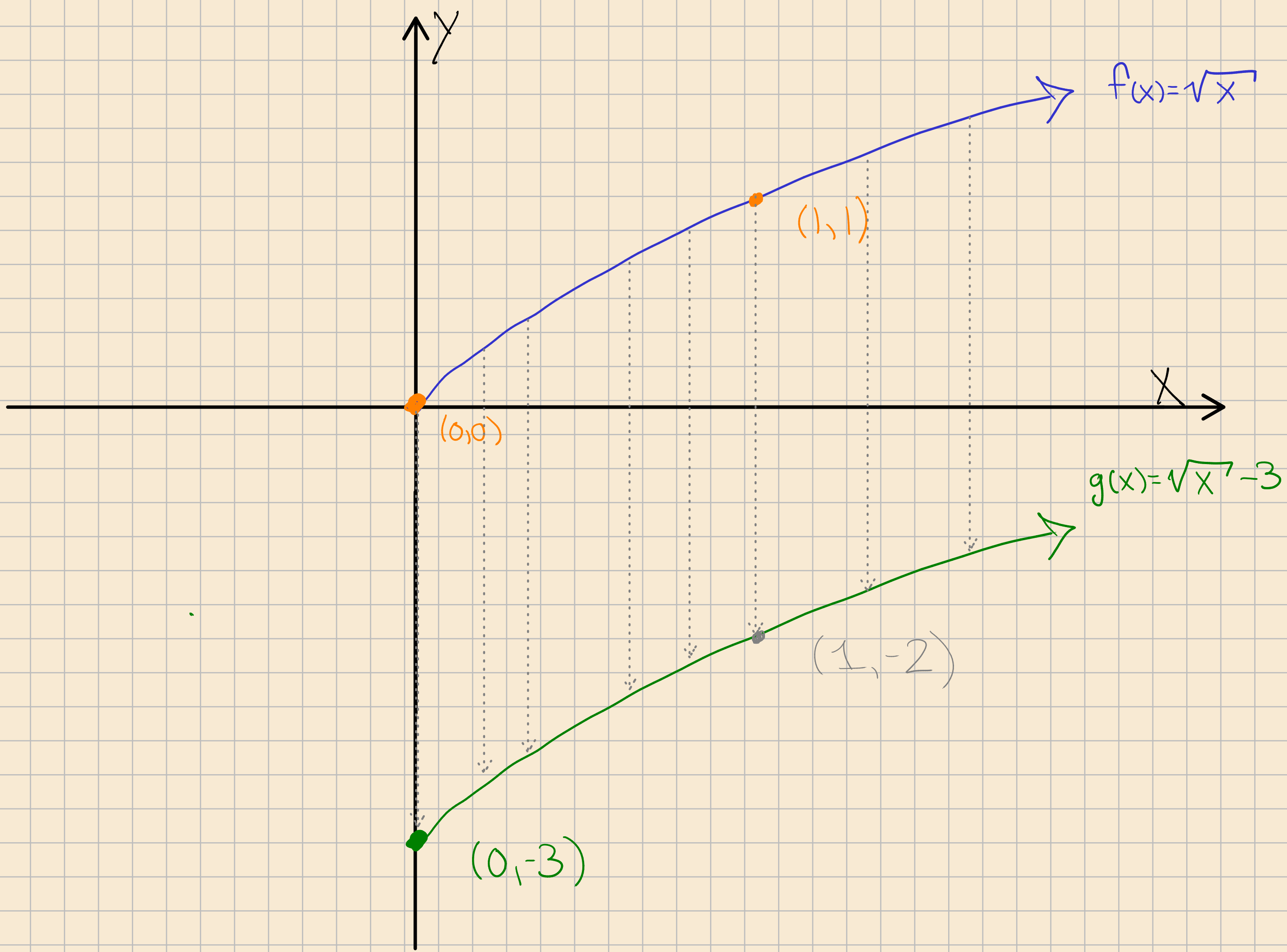


$\boxed{\Rightarrow \operatorname{dom}(f) = [0, \infty).}$



$\boxed{\Rightarrow \operatorname{Range}(f) = [0, \infty).}$

b) $f(x)-3$ shifts graph <u>down</u> by 3:



<u>Domain</u>: $\sqrt{x}-3$ still problematic for $x<0$,
or project:

$$\boxed{\text{domain}(g)=[0,\infty)}$$

<u>Range</u>: Project onto y-axis above to get

$$\boxed{\text{range}(g)=[-3,\infty).}$$

<u>Mnemonic</u>: $\text{range}(f(x)\pm c)=\underbrace{\text{range}(f)\pm c}_{\text{shift entire interval by } \pm c}$

$$\left([a,b]\pm c \underbrace{:=}_{\text{defined as}} [a\pm c, b\pm c]\right)$$

so " $[0,\infty)-3=[-3,\infty)$ ".