

Name: _____

You may use any resource you'd like for this quiz *except* for other people, i.e. your work must be your own. You can use calculators, notes, content on ELC, and general resources online, but you can not discuss work for this quiz with others. For calculations, you should show each step and the process used to arrive at that answer. Solutions without any work shown can not receive any credit.

- (1) (10 pts) Using equations and full sentences, give a definition of what it means for a function to be **continuous**.

Note: being able to draw the graph "without lifting your pencil" isn't a definition! Your definition should have something to do with limits.

A function f is continuous @ x_0 iff

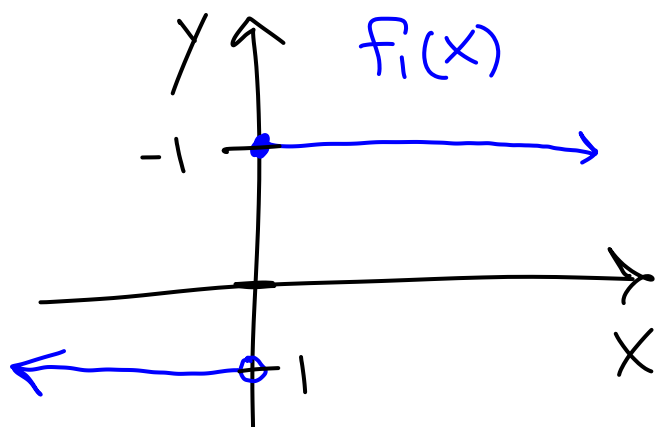
$$\lim_{x \rightarrow x_0} f(x) = f(x_0)$$

and is just continuous iff f is cts @ every x_0 in the domain of f .

- (2) (10 pts) Describe using equations or graphs two distinct functions that are not continuous, and use your definition from above to justify *why* they are not continuous.

Ex 1

$$f_1(x) = \begin{cases} +1, & x \geq 0 \\ -1, & x < 0 \end{cases}$$



Then

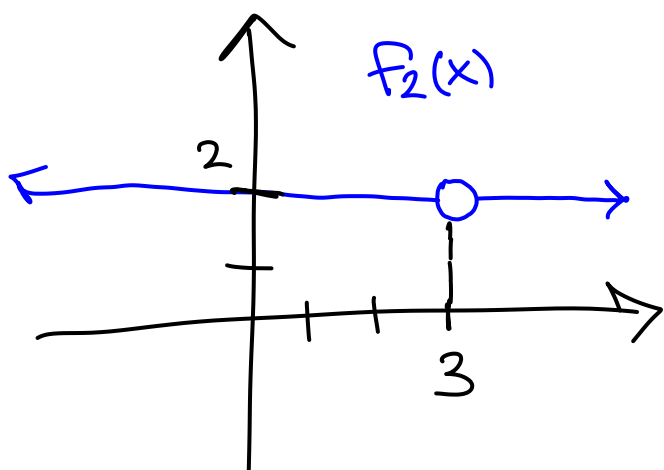
$$\lim_{x \rightarrow 0^+} f(x) = +1$$

$$\lim_{x \rightarrow 0^-} f(x) = -1$$

So $\lim_{x \rightarrow 0} f(x)$ DNE

$\Rightarrow f$ not cts @ $x_0 = 0$.

Ex 2



$$f(x) = \begin{cases} 2, & x \neq 3 \\ \text{undef}, & x = 3 \end{cases}$$

Then $x_0 = 3$ is not in domain (f), so f can't be cts @ $x_0 = 3$.

- (3) (10 points) Let f be the function defined by the formula $f(x) = \sqrt{5+x}$. Clearly state the difference quotient definition of the derivative function f , and compute the corresponding limit to determine an equation for $f'(x)$.

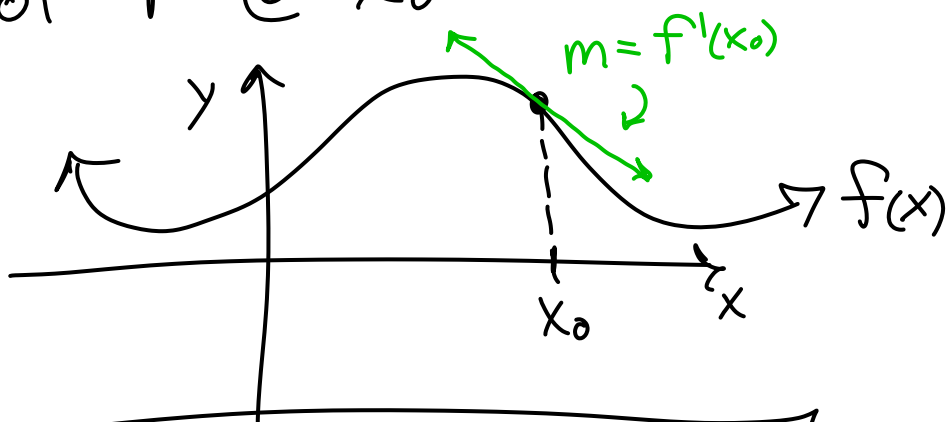
Hint: this should be a function of x , and should have something to do with limits.

$$\begin{aligned}
 f'(x) &:= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (\text{definition!}) \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{5+(x+h)} - \sqrt{5+x}}{h} \quad \left(\frac{\overbrace{\sqrt{5+(x+h)} + \sqrt{5+x}}^{=1}}{\underbrace{\sqrt{5+(x+h)} + \sqrt{5+x}}_{\text{conjugate}}} \right) \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{5} + \cancel{(x+h)} - (\cancel{5} + \cancel{x})}{h(\sqrt{5+(x+h)} + \sqrt{5+x})} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{5+(x+h)} + \sqrt{5+x}} \\
 &= \frac{1}{2\sqrt{5+x}}.
 \end{aligned}$$

- (4) (10 points) What is the slope of the tangent line of f at the point $x = \pi$? Justify your answer with at least one sentence.

Hint: this should be an actual real number. You should not need to calculate any additional limits for this part.

$f'(x_0)$ is the slope of the
tangent line of f @ x_0



$$\Rightarrow \text{Slope} = f'(\pi) = \frac{1}{2\sqrt{5+\pi}}$$

(5) (10 points, extra credit) Find and attach a picture of a cool mathematical object! Include a sentence or two on what it is or what it's supposed to represent. Here are some sites to get you started:

- (a) <https://homepage.univie.ac.at/herwig.hauser/bildergalerie/gallery.html>
- (b) <https://www.imaginary.org/galleries>
- (c) <https://silviana.org/gallery/hauser/>
- (d) <http://xahlee.info/surface/gallery.html>
- (e) <http://virtualmathmuseum.org>

Please enjoy ...

Camille!

