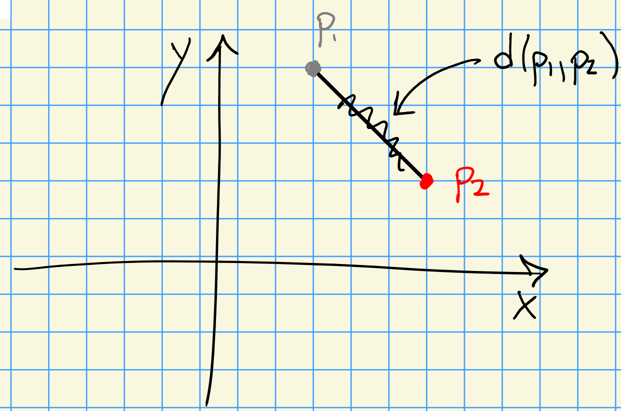
Quiz 1 (Solns).

What's the picture?



1. (5 points) Determine the distance between the points $P(-3, 2)$ and $Q(3, 5)$.

Distance formula: if

$$p_1 = (x_1, y_1)$$
$$p_2 = (x_2, y_2)$$

then

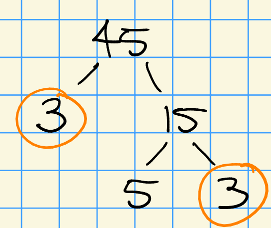
$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here

$$p_1 = (-3, 2)$$
$$p_2 = (3, 5)$$

$$\Rightarrow d(p_1, p_2) = \sqrt{(3-(-3))^2 + (5-2)^2}$$
$$= \sqrt{6^2 + 3^2}$$
$$= \sqrt{36 + 9}$$
$$= \sqrt{45}$$
$$= \sqrt{9 \cdot 5}$$
$$= \sqrt{9} \cdot \sqrt{5}$$
$$\boxed{= 3\sqrt{5}.}$$

Does 45 contain a perfect square?



Yes!

2. (5 points) Using the axes below make a sketch of the graph of the equation $2y + |x-1| = 3$.

$$2y + |x-1| = 3$$

$$\Rightarrow (2y + |x-1|) - (|x|-1) = 3 - (|x-1|)$$

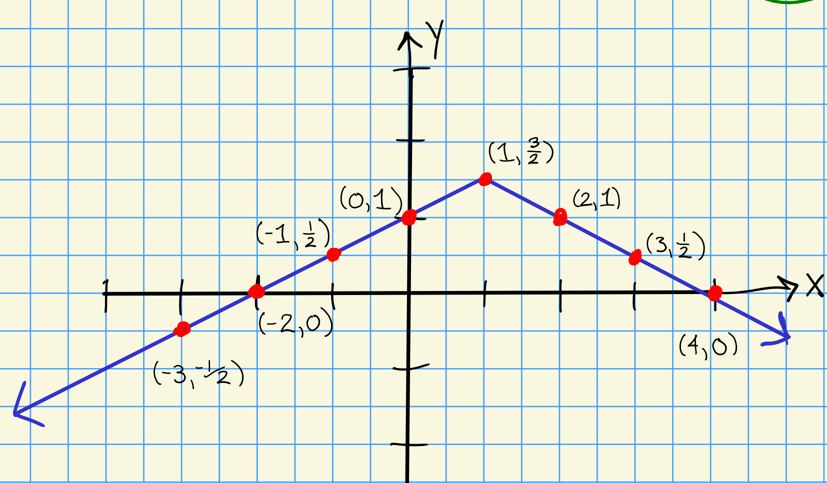
$$\Rightarrow \quad 2y = 3 - |x-1|$$

$$\Rightarrow \quad y = (1/2)(3 - |x-1|)$$

$$\Rightarrow \quad y = \frac{3}{2} - \frac{1}{2}|x-1|$$

Behavior of $|x|$ changes near $x=0$, so we should plot points around $|x-1|=0$

x	y
0	1
-1	1/2
+1	3/2
-2	0
+2	1
-3	-1/2
+3	1/2
+4	0



Short cut to avoid plotting points

- Parent Function: $y = |x|$, "changes direction" at $x=0$

$\Rightarrow$ $2y - |x-1| = 3$ "changes direction" at $x-1=0$, i.e. $x=1$

- $x=1 \Rightarrow 2y - |1-1| = 3$

$$\Rightarrow 2y = 3$$

$$\Rightarrow y = 1$$

So $\underline{(x,y)} = \underline{(1,1)}$ will be an important point.

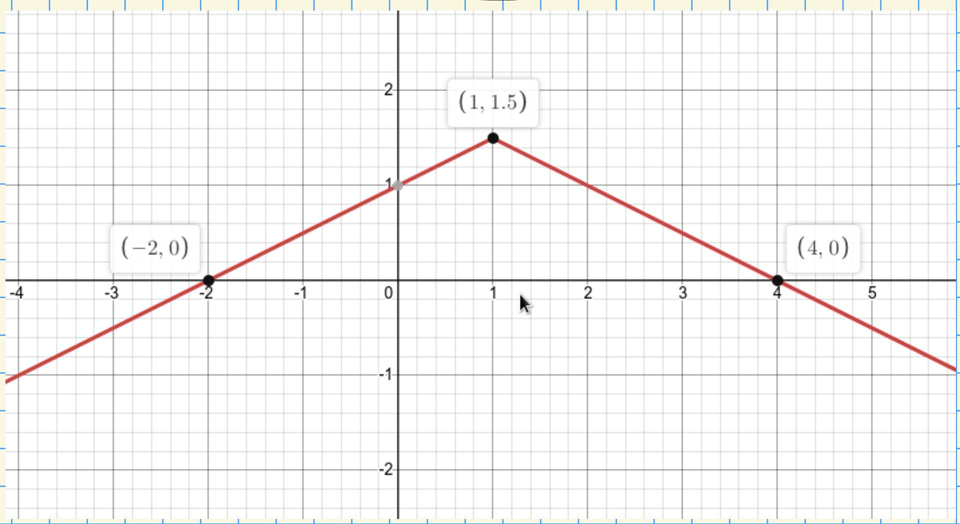
- Find intercepts:

x-intercept at $y=0$, so

$$2y - |x-1| = 3 \text{ and } y=0 \Rightarrow -|x-1| = 3$$
$$\Rightarrow |x-1| = 3$$
$$\Rightarrow x = 4, -2$$

Ask about this step in office hours if it's not immediately clear!

$\Rightarrow$ x-intercepts at $\underline{\underline{(4,0)}}$ and $\underline{\underline{(-2,0)}}$



3. (5 points) Determine all of the x-intercepts and y-intercepts of the graph of the following equation:

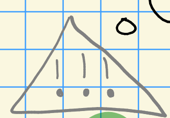
$$(x-\pi)^2 + \sqrt{1-\frac{y}{7}} = 9$$

- x-intercepts: set $y=0$.

$$(x-\pi)^2 + \sqrt{1-0} = 9$$
$$\Rightarrow (x-\pi)^2 + 1 = 9$$
$$\Rightarrow (x-\pi)^2 = 8$$
$$\Rightarrow x-\pi = \pm\sqrt{8}$$
$$\Rightarrow x = (\pm 2\sqrt{2}) + \pi \Rightarrow \text{intercepts at } \boxed{\begin{array}{c}(2\sqrt{2}+\pi, 0)\\ (-2\sqrt{2}+\pi, 0).\end{array}}$$

- y-intercepts: set $x=0$.

$$(0-\pi)^2 + \sqrt{1-(y/7)} = 9$$
$$\Rightarrow \pi^2 + \sqrt{1-(y/7)} = 9$$
$$\Rightarrow \sqrt{1-(y/7)} = 9-\pi^2$$



no real solution!

$$\Rightarrow 1-(y/7) = (9-\pi^2)^2$$
$$\Rightarrow 1-(9-\pi^2)^2 = (y/7)$$
$$\Rightarrow 7\left(1-(9-\pi^2)^2\right) = y$$
$$\Rightarrow y = 7-7(9-\pi^2)^2,$$

We'll continue the calculation, but note the subtle problem here: $\pi > 3 \Rightarrow \pi^2 > 3^2 = 9$, so $9-\pi^2$ is negative. But $\sqrt{\text{anything}}$ should be positive. This indicates that there won't be a real y-intercept.

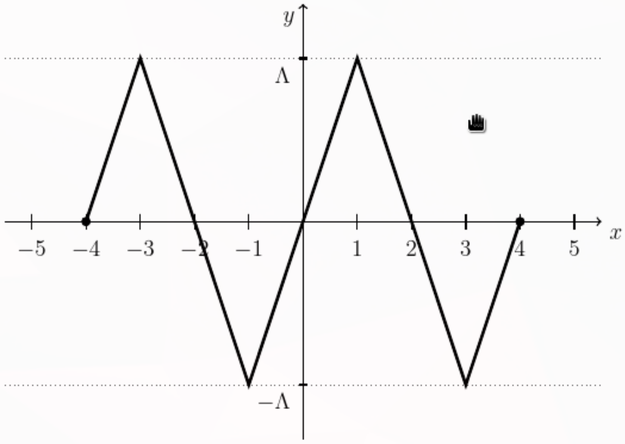
$$\boxed{\text{So "y-intercept" at } (0, 7-7(9-\pi^2)^2).}$$

(No actual y-intercept because of this issue

$$\boxed{\begin{array}{c}\text{Actual answer:}\\ \text{no y-intercept.}\end{array}}$$

Moral: "blind" algebra doesn't always work!

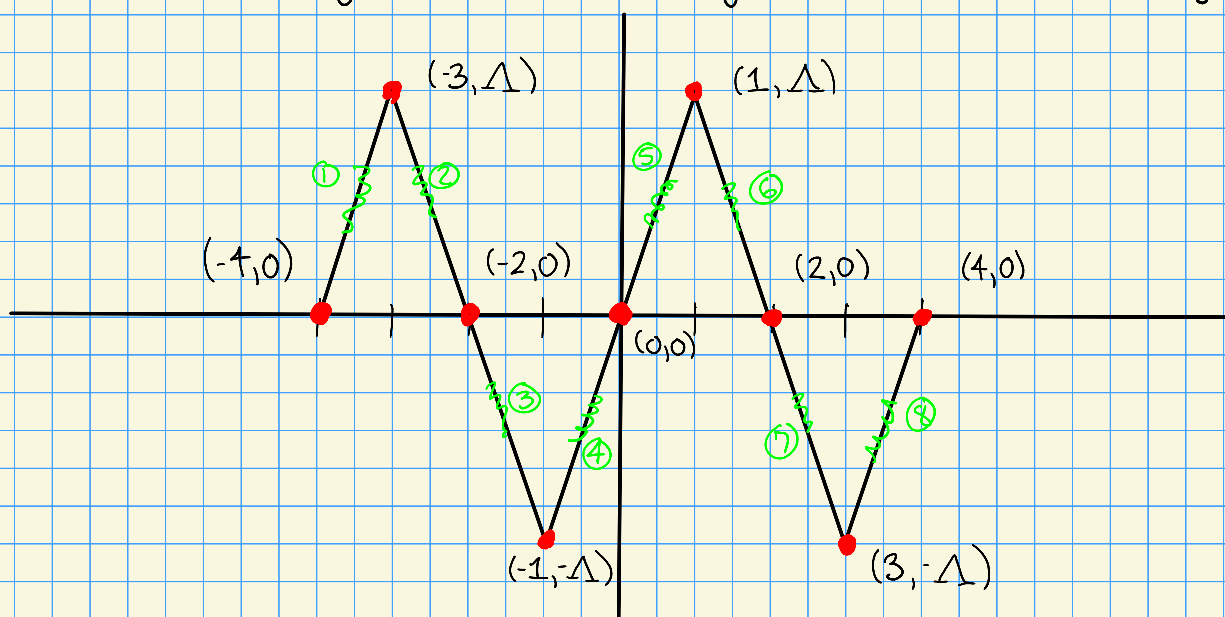
4. (5 points) Consider the curve pictured below, defined by a connected collection of straight line segments, where Λ is some unknown real number:



Write down a formula for the total length of this curve in terms of the unknown Λ.

Hint: break down the problem into smaller, easier pieces. Can you identify any points of the form (x, y) on this curve? Can you determine the length of any of the individual line segments in terms of Λ? How can you recombine all of these lengths to get a solution?

Break into 8 segments, compute length of each, and sum lengths.



Total Length = Sum of lengths ① through ⑧.

① $d_1 = \sqrt{(-3-(-4))^2 + (\Lambda - 0)^2} = \sqrt{1+\Lambda^2}$

② $d_2 = \sqrt{(-2-(-3))^2 + (0-\Lambda)^2} = \sqrt{1+(-\Lambda)^2} = \sqrt{1+\Lambda^2}$

③ $d_3 = \sqrt{(-1-(-2))^2 + (-\Lambda-0)^2} = \sqrt{1+(-\Lambda)^2} = \sqrt{1+\Lambda^2}$

④ $d_4 = \sqrt{(0-(-1))^2 + (0-(-\Lambda))^2} = \sqrt{1+\Lambda^2}$

⑤ $d_5 = \sqrt{(1-0)^2 + (\Delta - 0)^2} = \sqrt{1+\Delta^2}$

⑥ $d_6 = \sqrt{(2-1)^2 + (0-\Delta)^2} = \sqrt{1+(-\Delta)^2} = \sqrt{1+\Delta^2}$

⑦ $d_7 = \sqrt{(3-2)^2 + (-\Delta-0)^2} = \sqrt{1+(-\Delta)^2} = \sqrt{1+\Delta^2}$

⑧ $d_8 = \sqrt{(4-3)^2 + (0-(-\Delta))^2} = \sqrt{1+\Delta^2}$

Since this gives 8 segments, each of length $\sqrt{1+\Delta^2}$,

$$\boxed{\text{Total Length} = 8\sqrt{1+\Delta^2}.}$$