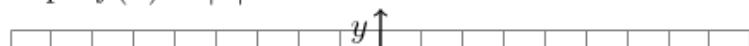


1.6.a Solns

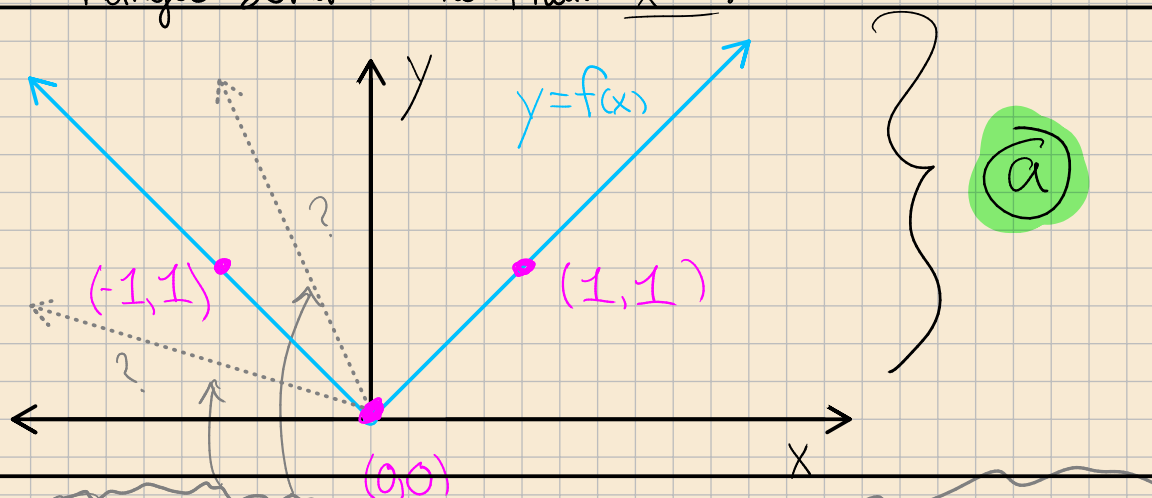
1. (3 points) For this problem, let $f(x) = |x|$.

(a) Graph $f(x) = |x|$. Then determine the domain and range of $f(x)$.



(b) Graph and label $f(x+2)$ on the coordinate system above. Then determine the domain and range of $f(x+2)$.

What are some "interesting" points? Think about where the function changes behavior: here, near $x=0$.



Need 3 points, because 2 points only determine a single line! Without a 3rd point, we don't know which of these possibilities happens.

Domain and Range

Hope: both are all of $\mathbb{R} = (-\infty, \infty)$. What are the problematic points? None! For all $x \in (-\infty, \infty)$, $f(x) = |x|$ makes sense.

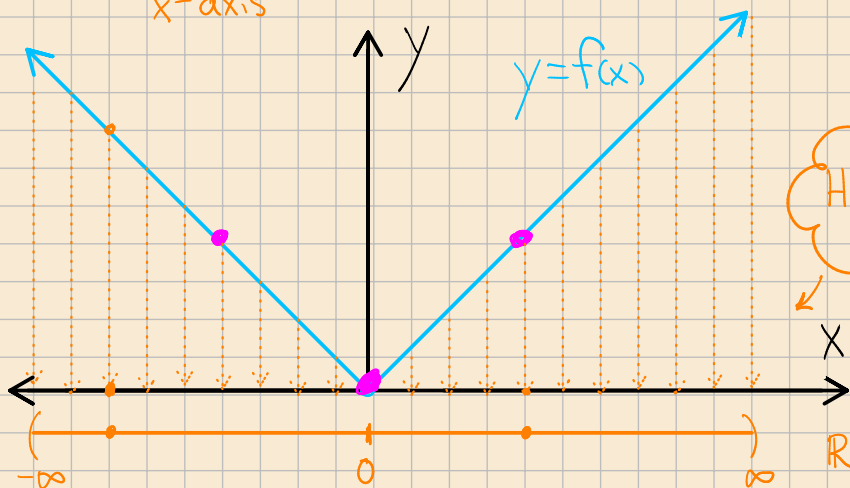
$$\Rightarrow \text{domain}(f) = \mathbb{R} = (-\infty, \infty)$$

Range: use the fact that $|x| \geq 0$ for all $x \in \text{dom}(f)$, so

$$\text{range}(f) = [0, \infty).$$

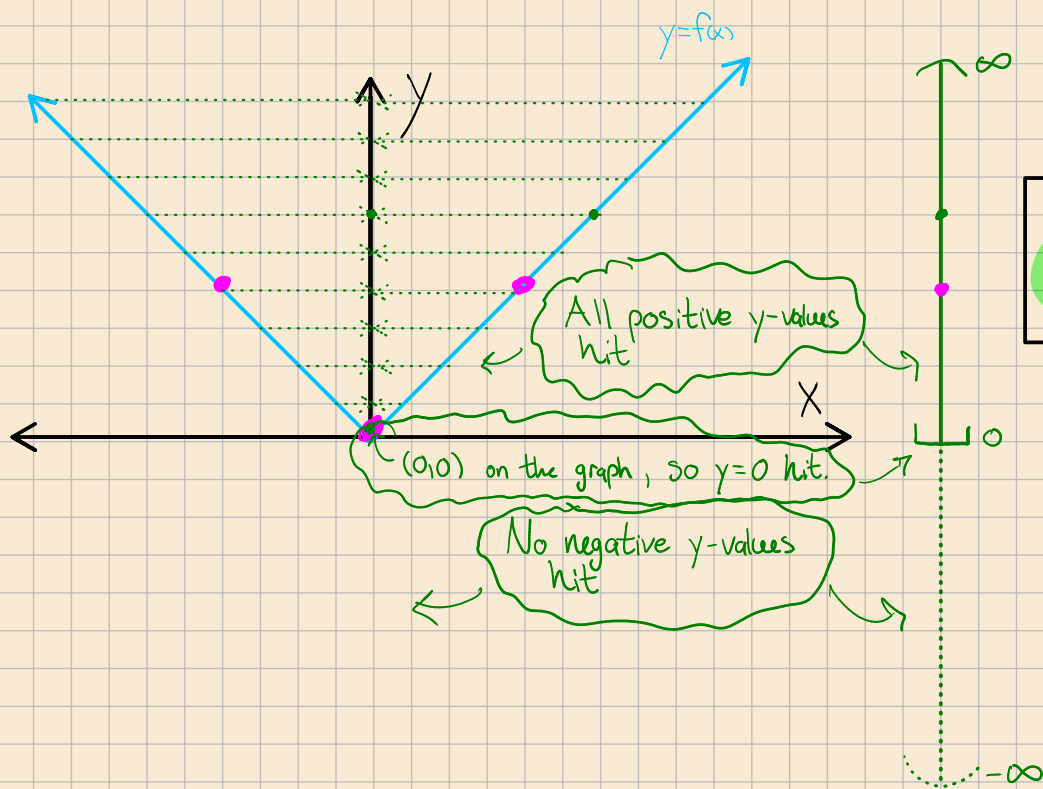
Alternatively, the projection method:

Domain: project to
x-axis



Hits every x value

$\Rightarrow \text{dom}(f) = \mathbb{R}.$



All positive y-values hit

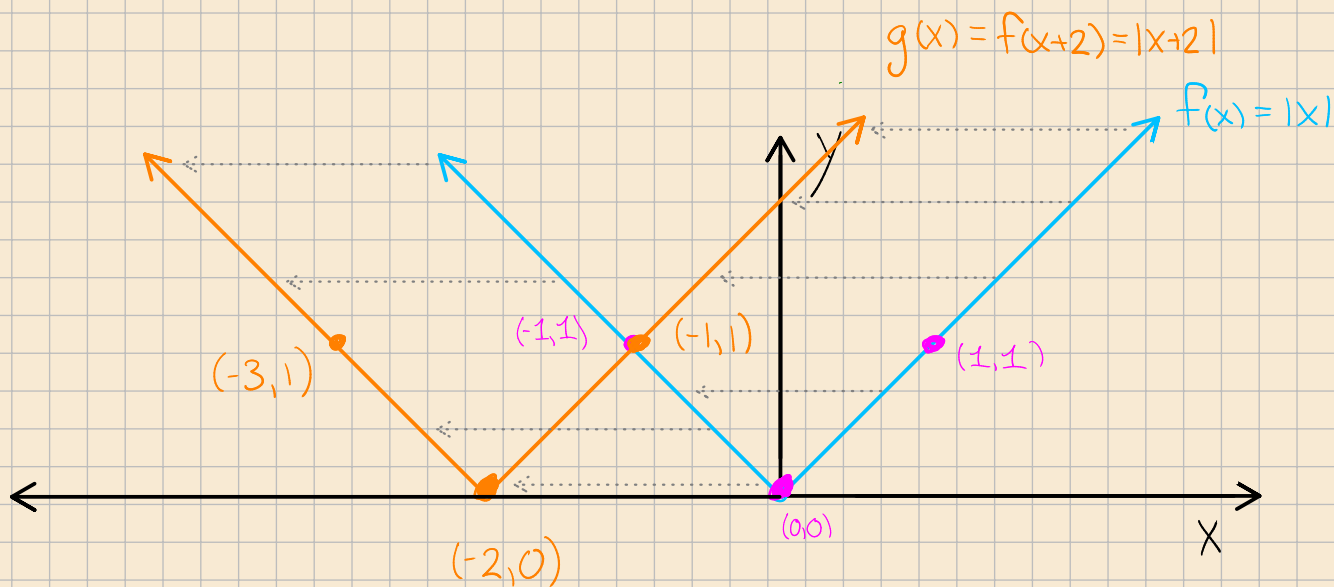
(0,0) on the graph, so $y=0$ hit.

No negative y-values hit

$\Rightarrow \text{range}(f) = [0, \infty).$

(b) Tricky: Shifts inside a function have "opposite" effects

- $f(x)+2$: Shifts graph up by 2
- $f(x+2)$: Shifts graph left by 2

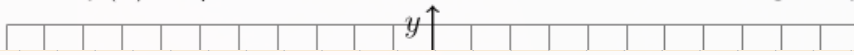


But in this instance, note that this doesn't change the x or y projections.

$$\Rightarrow \text{domain}(g) = \mathbb{R} = (-\infty, \infty),$$
$$\text{range}(g) = [0, \infty).$$

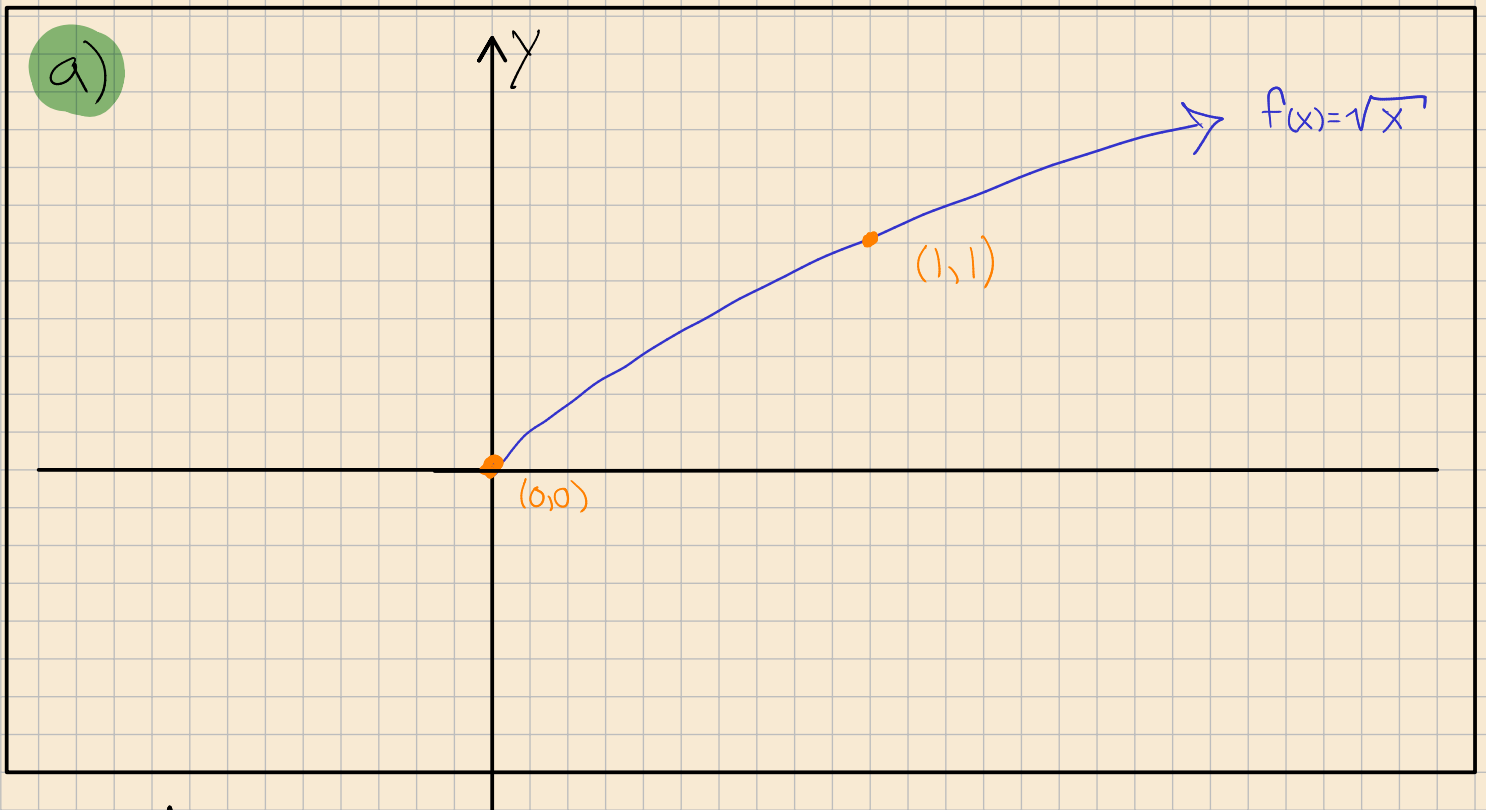
2. (3 points) For this problem, let $f(x) = \sqrt{x}$.

(a) Graph $f(x) = \sqrt{x}$. Then determine the domain and range of $f(x)$.



(b) Graph and label $f(x) - 3$ on the coordinate system above. Then determine the domain and range of $f(x) - 3$.

What are "interesting" points? The function f changes behavior at $x=0$, namely it is not defined as soon as $x < 0$.



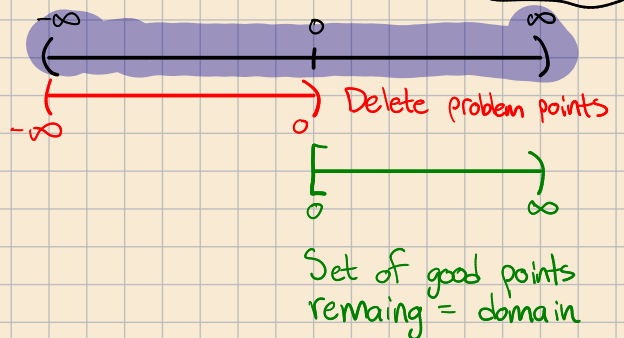
Domain & Range:

Domain: Hope it's all of \mathbb{R} , but what x -values are problematic?

Answer: any $x < 0$ (note $x=0$ is okay.)

Hope: all of \mathbb{R}

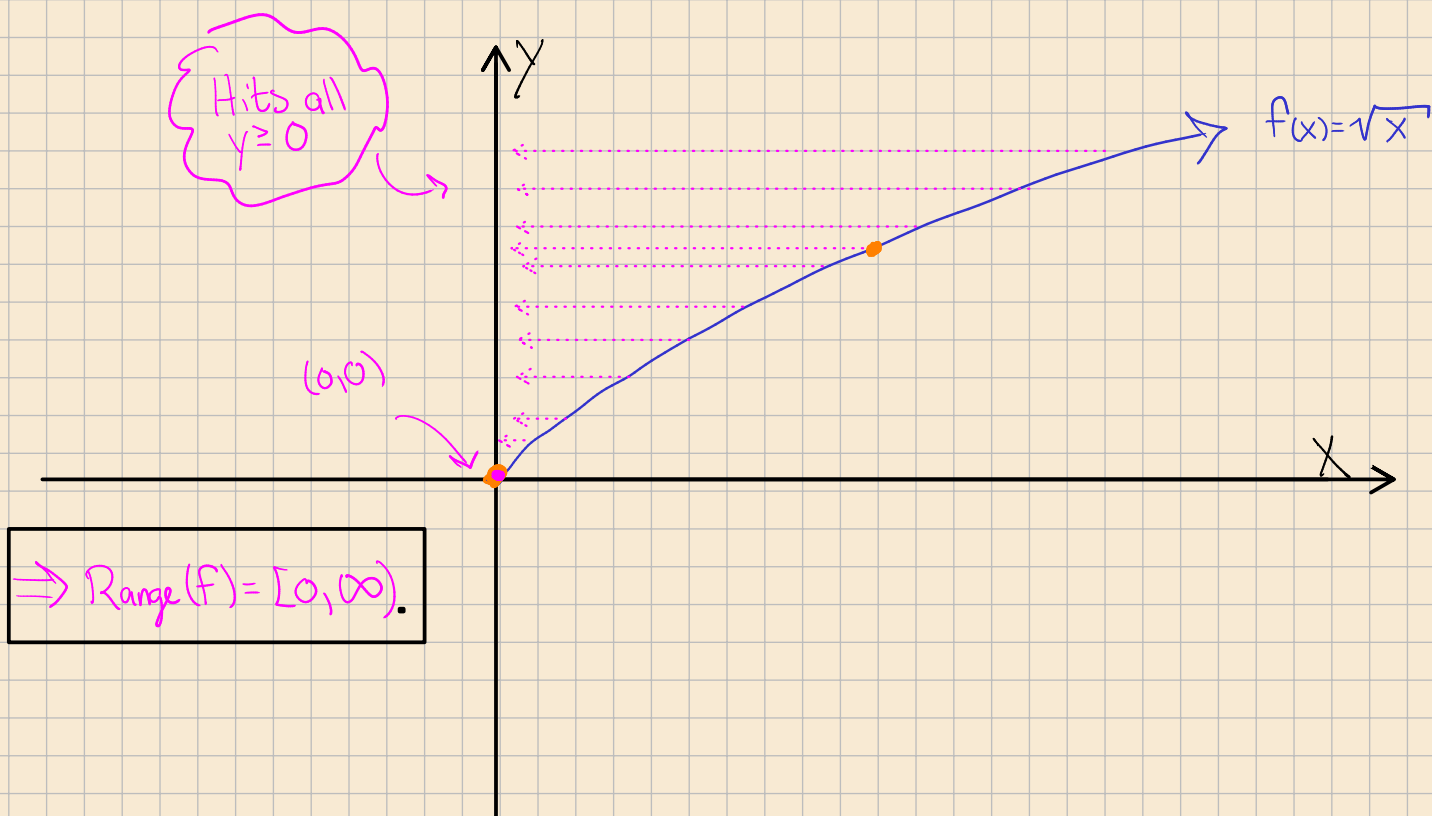
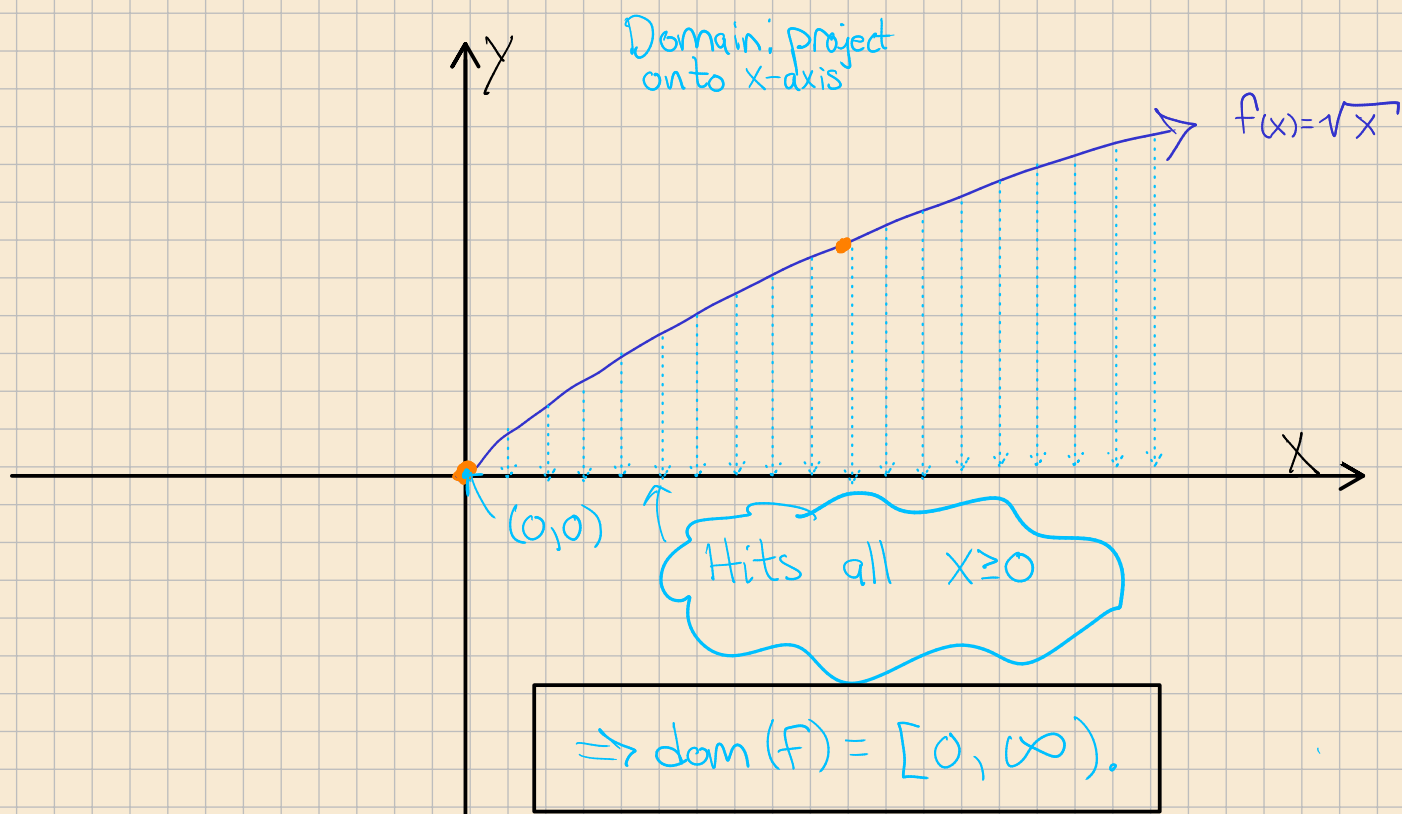
$$\begin{aligned} \text{So domain}(f) &= \mathbb{R} \setminus \{x \in \mathbb{R} \mid x < 0\} \\ &= (-\infty, \infty) \setminus (-\infty, 0) \\ &= [0, \infty). \end{aligned}$$



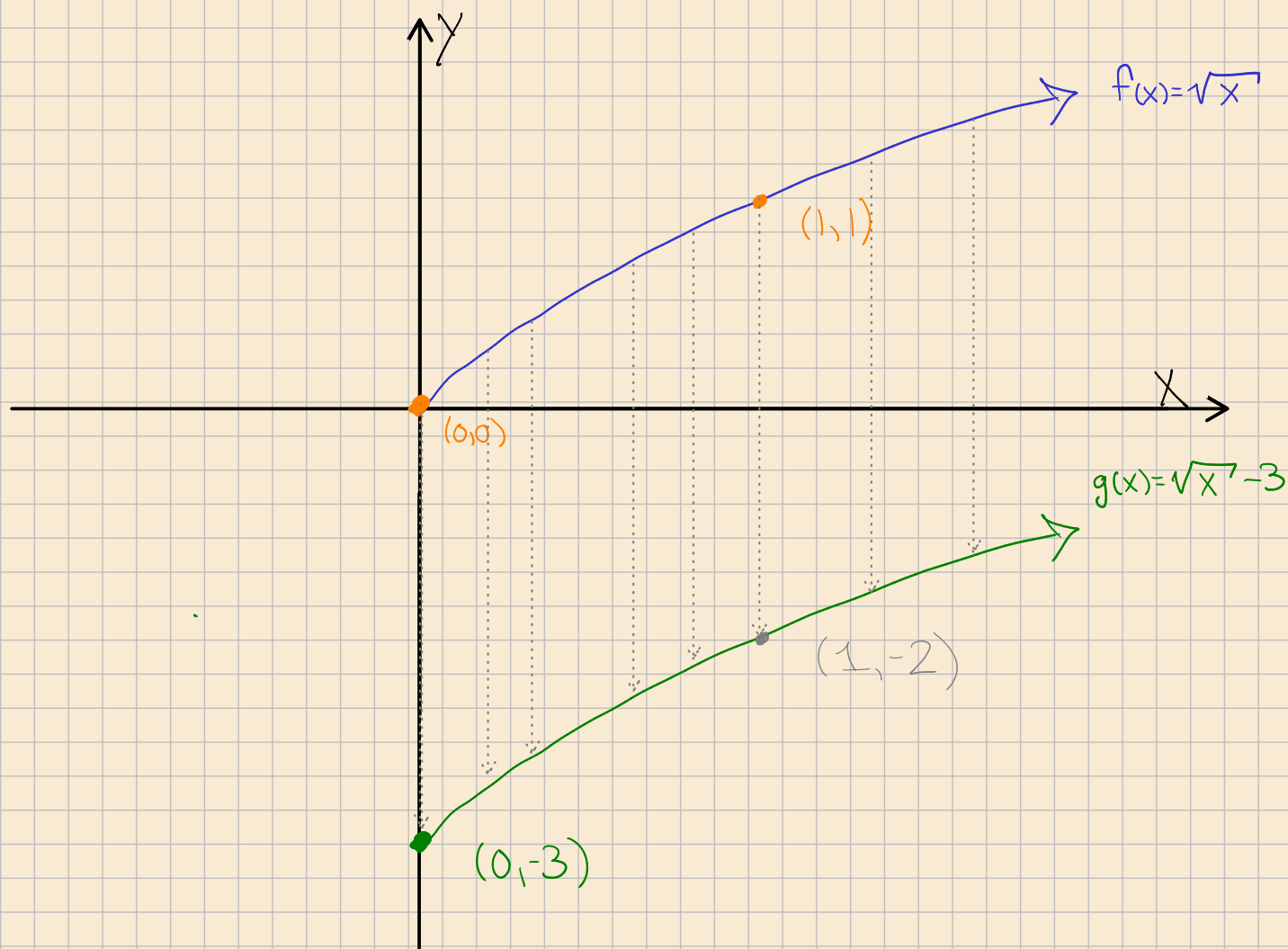
Range: Use the fact that $\sqrt{x} \geq 0$ for all x in $\text{domain}(\sqrt{x})$

$$\Rightarrow \text{range}(f) = [0, \infty).$$

Alternatively, we can project:



b) $f(x)-3$ shifts graph down by 3:



Domain: $\sqrt{x} - 3$ still problematic for $x < 0$,
or project:

$$\text{domain}(g) = [0, \infty)$$

Range: Project onto y-axis above to get

$$\text{range}(g) = [-3, \infty)$$

Mnemonic: $\text{range}(f(x) \pm c) = \text{range}(f) \pm c$

shift entire interval by $\pm c$

$$\left([a, b] \pm c := \underbrace{[a \pm c, b \pm c]}_{\text{defined as}} \right)$$

So " $[0, \infty) - 3 = [-3, \infty)$ ".