Title

Contents

0.1 Algebra

- Looking at real roots:
  - Let p be number of sign changes in $f(x)$;
  - Let q be number of sign changes in $f(-x)$;
  - Let n be the degree of f.
  - Then p gives the maximum number of positive real roots, q gives the maximum number of negative real roots, and $n - p - q$ gives the *minimum* number of complex roots.
  - Rational Roots Theorem: If $p(x) = ax^n + \cdots + c$ and $r = \frac{p}{q}$ where $p(r) = 0$, then $p \Big| c$ and $q \Big| a$.
- Properties of logs:
  - $\ln(\prod) = \sum \ln$ but $\prod \ln \neq \ln \sum$
  - $\log_b x = \dfrac{\ln x}{\ln b}$

Be careful! $\dfrac{\ln x}{\ln y} \neq \ln \dfrac{x}{y} = \ln x - \ln y$

- Completing the square:
  - $p(x) = ax^2 + bx + c \implies p(x) = a(x + \dfrac{b}{2a})^2 + -\dfrac{1}{2}\left(\dfrac{b^2 - 4ac}{2a}\right)$

0.2 Geometry

- Generic Conic Sections

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$\frac{(x - x_0)^2}{w_0} \pm \frac{(y - y_0)^2}{h_0} = c$$

- Circles:

$$Ax^2 + By^2 + C = 0 \qquad\qquad (x - x_0)^2 + (y - y_0)^2 = r^2$$

  - Defining trait: locus of points at a constant distance from the **center**
  - **Center** at (x_0, y_0)

- Parabolas:

$$Ax^2 + Bx + Cy + D = 0 \qquad\qquad y = ax^2$$

  - Defining Trait:
    - ♢ Locus of points equidistant from the **focus** (a point) and the **directrix** (a line)
    - ♢ #todo add image
  - **Focus** at $(0, \frac{1}{4a})$
  - **Directrix** at the line $y = -\frac{1}{4a}$
    - ♢ For an arbitrary quadratic: complete the square to write in the form $y = a(x - w_0)^2 + h_0$, and translate points of interest by by $(x + w_0, y + h_0)$

- Ellipses:

$$\frac{x^2}{w^2} + \frac{y^2}{h^2} = 1$$

  - Defining trait:
    - ♢ The locus of points where the *sum* of distances to two **focii** are constant.
  - **Center** at $(0, 0)$ (can translate easily)
  - **Vertices** at $(\pm w, 0)$ and $(0, \pm h)$
  - **Focii** at $F_1 = (\sqrt{w^2 - h^2}, 0), F_2 = (-\sqrt{w^2 - h^2}, 0)$
  - Another useful shortcut form:

- Hyperbolas:

$$\frac{x^2}{w^2} - \frac{y^2}{h^2} = 1$$

  - Defining trait:
    - ♢ Locus of points where the *difference* between the distances to two **focii** are constant.
  - **Vertices** at $(0, \pm h)$ and $(\pm w, 0)$
  - **Focii** at $F_1 = (\sqrt{w^2 + h^2}, 0), F_2 = (-\sqrt{w^2 + h^2}, 0)$

- Summary of Traits:

  - One point p:
    - ♢ Distance to p is constant: circle
  - Two points a, b:
    - ♢ Distance to a equal to distance to b equals a constant: a line bisecting the midpoint of the line connecting them
    - ♢ Difference of distances constant: ellipse
    - ♢ Sum of differences constant: hyperbola
  - Point p and a line l:
    - ♢ Distance to p equals distance to l equals a constant: parabola

- Areas of certain figures: