**Math 8190: Representations of Algebraic Groups, Fall 2020 (Nakano)**

A major problem in representation theory has been the determination of a character formula and the dimensions of finite dimensional simple modules over a connected reductive algebraic group $G$. Lusztig conjectured that the characters could be computed recursively in terms of "Kazhdan-Lusztig" polynomials. These polynomials arise from the Hecke algebra associated with the Weyl group of the given algebraic group. It turns out that this conjecture is true for very large primes. In 2013, Williamson showed that there are infinitely many counterexamples to the original conjecture.

One of the goals of this course is to introduce the terminology and ideas used in formulating the Lusztig conjecture. For this purpose most of the material will be taken from Jantzen's "Representations of Algebraic Groups".

In the first part of the course we will introduce the basic notions in representation theory and classify the finite dimensional simple modules for a reductive algebraic group $G$. This will entail using the geometry of the flag variety $G/B$ and looking at global sections of line bundles for $G/B$. Later we will discuss classical theorems involving these G-modules such as Kempf's vanishing theorem, the Bott-Borel-Weil theorem, and Weyl's Character Theorem.

Additional topics may include, cohomology and representations of Frobenius kernels $G_r$ and related module categories $G_rT$ and $G_rB$, Strong Linkage Principle, injective $G_r$ modules, Steinberg's twisted tensor product formula, and the Translation Principle.

In this course we will use an interesting mix of tools from algebra, topology and algebraic geometry.

Recommended reading list:

J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer-Verlag, 1970.

J.C. Jantzen, Representations of Algebraic Groups, American Math. Soc., 2004.

W.C. Waterhouse, Introduction to Affine Group Schemes, Springer-Verlag, 1979.