Review Excercises → Discuss / Present 8th Sept Tuesday

1) Show that $S^n \subset \mathbb{R}^{n+1}$ is a smooth manifold.
(did $n=1$, do $n>1$)

2) $\square$ is a smooth manifold

Really: If X is a smooth manifold and M is a topological manifold homeo to X, give M a C^∞ structure.

3) If $M \cong N$ (diffeomorphism)

show $\dim(M) = \dim(N)$

Ques: Is there a non-alg-top proof in smooth setting?

Note: Dim. of Manifold = dim of target of charts.

Is this well defined?

Same issue: Boils down to:

if $\Phi : U \longrightarrow V$

where $U \underset{\text{open}}{\subset} \mathbb{R}^n$ & $V \underset{\text{open}}{\subset} \mathbb{R}^m$

is C^∞, inverse is C^∞,

prove $n = m$.

(Invariance of Domain)

4. Prove an open subset of manifold is a manifold.

5. If M is a smooth manifold with chart (U, φ_U)
$\varphi_U : U \longrightarrow \mathbb{R}^m$

1) prove φ_U is a smooth map.

2) prove φ_U is a diffeomorphism from U to $\varphi_U(U)$.

6. $\mathbb{R}$, $U = \mathbb{R}$, $\varphi_U : x \longmapsto x^3$

& $V = \mathbb{R}$, $\varphi_V : x \longmapsto x$

Show that $\{(U, \varphi_U), (V, \varphi_V)\}$ is not a smooth atlas.

7. Given def. of submanifolds, prove that submanifolds are manifolds.

8. $\mathbb{R}P^n = S^n / \vec{x} \sim -\vec{x}$

Describe "natural" atlas making $\mathbb{R}P^n$ a smooth manifold such that

$$q : S^n \longrightarrow \mathbb{R}P^n \text{ is smooth.}$$

9. Fill in the gaps in last lecture (Tangent Space of a Manifold, 27th Aug lecture)

Not on Tuesday, later in the class.