$\boxed{\text{Setup}}$

- $X = \mathbb{P}^1/k$, $\quad k = \mathbb{F}_q$, $\quad \mathrm{char}\, k = p$

- $F = k(X)$, $\mathcal{O}_x$ valuation rings for $x \in |X|$

- $S = \{0, \infty\} \subseteq |X|$, $\quad G = Sp(V)$

- Fix a symplectic basis of V:

$$\{\underbrace{e_1, \cdots, e_n}_{0}, \overbrace{e_{-n}}^{\pm 1}, \cdots, e_{-1}\}, \qquad \begin{array}{l} \langle e_i, e_{-j} \rangle = 1 \\ \langle e_{\pm i}, e_{\pm j} \rangle = 0 \end{array}$$

- Choose level structures

t unif @ $x=0$ $\qquad\qquad$ τ unif @ ∞

$$K_0 = \left\{ \left[\begin{array}{c|c} \mathcal{O}_0 & t\mathcal{O}_0 \\ \hline \mathcal{O}_0 & \mathcal{O}_0 \end{array} \right] \right\}, \qquad K_\infty = \left\{ \left[\begin{array}{c|c} I_2 + \tau\mathcal{O}_\infty & \mathcal{O}_\infty \\ \hline \tau\mathcal{O}_\infty & I_2 + \tau\mathcal{O}_\infty \end{array} \right] \right\}$$

2x2

In the above basis

$$\Rightarrow \mathrm{Bun}_{Sp_{2n}}(K_0, K_\infty) = \left\{ \begin{array}{ll} (V, &] \text{ Vect. bun, rk } 2n \\ \omega, &] \text{ Symp. form} \\ L_0 \subseteq V_0 & \\ L_\infty \subseteq V_\infty &] \text{ Lagrangian} \\ \{\ell_1, \cdots, \ell_n\}) &] \text{ basis for } L_\infty \end{array} \right\}$$

Z

Next: Peter