Rating guide:

- ★☆☆☆☆: Designed for students who have taken high-school Calculus, and have completed or are co-enrolled in a college Calculus sequence.
- ★★☆☆☆: Designed for students who have completed the Calculus sequence, and have completed or are co-enrolled in lower-division linear algebra and differential equations.
- ★★★☆☆: Designed for students who have completed a course in proofs and are entering or co-enrolled in introductory upper-division mathematics courses: sequences and series, real analysis, abstract algebra, upper division linear algebra, number theory, etc.
- ★★★★☆: Designed for advanced students who are well-practiced in proofs and are taking (or have taken) advanced upper-division mathematics courses (e.g. a full sequence in analysis or algebra, or courses like complex analysis, differential geometry, point-set topoloy)
- ★★★★★: Designed for advanced students who have completed several upper division courses and are prepared for introductory graduate-level material (at the level of qualifying exam courses, although such material is not assumed).

Generatingfunctionology

- **Texts:**
  - Grimaldi, *Discrete and Combinatorial Mathematics*. Wilf, *Generatingfunctionology*. Bona, *A Walk Through Combinatorics*.
- **Difficulty / Background Required:**
  - Difficulty: ★★☆☆☆
  - Familiarity with: Calculus (power series, Taylor expansion, differentiation/integration). Discrete mathematics useful but not required.
- **Project outline:**

  This is a very hands-on, computational project in combinatorics. A proposed goal is to find and compute generating functions for several interesting sequences such as Stirling numbers (of the first and second kinds) and particular types of integer partitions enumerated by Young Tableaux. Along the way, this will involve learning a techniques for more basic counting problems and learning about well-known sequences such as binomial and multinomial coefficients, Catalan numbers, integer compositions, and integer partitions. We'll also study a general technique for solving recurrence relations using generating functions. Time permitting, this can lead into the arithmetic variants of generating functions: Gauss sums, zeta functions, Dirichlet series, and L-functions.
- **Notes:** Any type of combinatorics or counting problems are fair game for this project, so there's room follow students' interests here.

Category Theory

- **Texts:**
  - Mac Lane, *Categories for the Working Mathematician*. Rhiel, *Category Theory in Context*.
- **Difficulty / Background Required:**

- Difficulty: ★★★☆☆
- Familiarity with: Abstract algebra (groups, rings, fields, modules if possible), some topology and/or algebraic geometry useful but not required.

- **Project outline:**

  This project will be to understand many common constructions in category theory, a language which is widely used in modern mathematics but is not (usually) taught explicitly in the standard undergraduate or graduate curriculum. The goal will be to understand a proof of the Yoneda lemma, a result on the representability of functors which forms the cornerstone of 20th century algebraic geometry.

- **Notes:**

  The actual content we'd cover in this project is not incredibly technical or difficult, but *can* be very abstract and formal if the student doesn't have background to draw upon for intuition. However, we'll introduce basic examples from all of these fields along the way, so this can be a good "survey" project for students who want some exposure to a variety of areas (i.e. those where category theory shows up).

Homotopy Theory and Algebraic Topology

- **Texts:**
  - Hatcher, *Algebraic Topology*. Griffiths and Morgan, *Rational Homotopy Theory and Differential Forms*. Bott and Tu, *Differential Forms in Algebraic Topology*.
- **Difficulty / Background Required:**
  - Difficulty: ★★★★☆
  - Familiarity with: Point-set topology, abstract algebra (groups and rings), linear algebra (abstract vector spaces).
- **Project outline:**

  This is a project in homotopy theory, the subject I myself studied as an undergraduate! An outstanding problem in the field is the computation of homotopy groups of spheres, so the goal of the project will be to understand what these objects are, what algebraic structure they have, and to explicitly compute some known groups in small dimensions. Along the way, we'll work to understand how to use some standard computational tools from algebraic topology. Time permitting, we'll discuss a old but major result in the field which classifies all of these groups when we work over the rational numbers instead of the integers.

- **Notes:** If students have a significant interest in topology, this project can branch in several different directions. E.g. stable homotopy is a natural follow-up and a major research area for those who prefer the algebraic side. Cobordism and TQFTs (Topological Quantum Field Theories) are another direction that may be of interest to those who like pictorial/diagrammatic results or applications to physics.

Complex Manifolds and Hodge Theory

- **Texts:**

    - Griffiths and Harris, *Complex Algebraic Geometry*. Reid, *Undergraduate Algebraic Geometry*. Shafarevich, *Basic Algebraic Geometry 1*.
- **Difficulty / Background Required:**
    - Difficulty: ★★★★★
    - Familiarity with or willingness to learn: Abstract algebra (kernels, images), linear algebra (dimension, orthogonality, direct sums, diagonalization), calculus on smooth real manifolds (differential forms, Stokes' theorem), very basic algebraic topology (cohomology).
- **Project outline:** This is primarily a project in (complex) algebraic geometry, with some strong ties to (complex) manifolds and differential geometry. A proposed goal is to reach an understanding of the *Hodge decomposition* for complex manifolds, the corresponding *Hodge diamond* – a highly symmetric collection of numbers attached to complex manifolds – and to compute the diamond in a concrete example such as an elliptic curve (given the structure of a complex torus). For students interested in physics, these structures play a prominent role in *string theory*, an enhanced version of spacetime in which there are an extra 6 dimensions occupied by Calabi-Yau manifolds. These come in pairs which exhibit *mirror symmetry*, which can be seen as a certain "flip" in the Hodge diamond.
- **Notes:** This is close to the topic of my own research, so this is closest to my expertise. Getting through this *entire* project in one term may only be a reasonable expectation for advanced students who have seen some of the background material elsewhere – however, some things can be black-boxed, and this could make for a reasonable two-term (Summer + Fall) project for students who want to pick up the necessary background material along the way. Alternatively, it would also be perfectly reasonable to just pick a smaller piece of this project to focus on, e.g. just learning about introductory topics in complex manifolds, algebraic geometry, algebraic topology, etc – whatever strikes your fancy!

Other

The following are project ideas I would be willing to go through alongside any students interested:

- Algebra: group theory, ring theory, linear algebra, introductory homological algebra, commutative algebra, fields and Galois theory, modules, etc.
    - Possible references:
        - Dummit and Foote's *Algebra*
        - Allufi's *Algebra Chapter Zero*.
- Number theory: modular arithmetic, cryptography, basics up to Legendre symbols and quadratic reciprocity, algebraic number theory (quadratic number fields, class numbers, split/ramified/inert primes, etc), or diophantine equations.
    - Possible references:
        - Leveque's *Fundamentals of Number Theory*,
        - Apostol's *Introduction to Analytic Number Theory*,
        - Milne's *Algebraic Number Theory*.
- Topology: smooth manifolds, knot theory, Morse theory, symplectic topology, Floer homology.
    - Possible references:
        - Lee's *Smooth Manifolds*,

        - Audin-Damian's *Morse Theory and Floer Homology*,
        - Roberts' *Knotes* (on knot theory),
        - Cannas da Silva's *Lectures on Symplectic Geometry*.
- Algebraic geometry: affine varieties, algebraic curves, moduli spaces (e.g. of elliptic curves, with relations to modular forms), basics of sheaves and/or schemes.
    - Possible references:
        - Reid's *Undergraduate Algebraic Geometry*,
        - Cox's *Ideals Varieties and Algorithms*.
- Representation theory of finite groups, introductory theory of Lie algebras
    - Possible references:
        - Dummit and Foote's *Algebra*,
        - Fulton-Harris' *Representation Theory: A First Course*,
        - Humphreys' *Introduction to Lie Algebras and Representation Theory*.