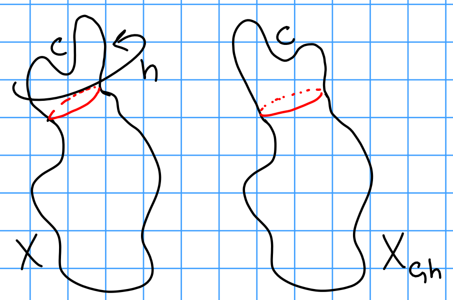
Let X^4 be a smooth, closed, 1-connected 4-mfd.

$$\mathcal{M}_X = \{Y \text{ homeo to } X\} / \text{Diffeo} \sim \text{Collection of smooth structures}$$

($\mathcal{M}_{S^4}$ unknown! Speculated $\mathcal{M}_X$ always infinite for X^4 but always finite $n \neq 4$)

Cork: (C, h) — h: diffeo $\partial C \circlearrowleft$; C: Compact contractible 4-mfd

Cork twist: $X_{C,h} \cong (X \setminus C) \bigsqcup_h C \underbrace{\cong_{Top} X}_{\text{Friedman's result}}$



Defns: (C,h) is

- **Trivial** if h extends to a diffeo $C \circlearrowleft$
- **Finite** if order$(h) < \infty$ (order 2 ~ involutive)
- **Simple** if $C = h^0 \cup \underbrace{h^1_i}_{\text{One handles}} \cup \underbrace{h^2_j}_{\text{2-handles}}$ where $\pi_1 C = \langle x_1, \cdots, x_k \mid r_1, \cdots r_k \rangle$ and the h^2_j's homotopically cancel the h^1_i's by handle slides?
  ("AC" condition)
  - $C \bigsqcup_h C \cong S^4$

Example of a nontrivial cork (Akbulut 91)

Use Kirby Calculus to show it's simple, see "dot diagram" for involution.

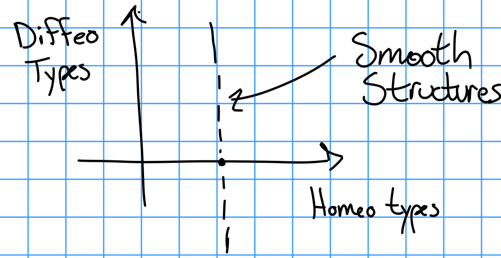
CT: $\forall Y \in \mathcal{M}_X$, $\exists (C \subseteq X, h)$ s.t. $Y \cong X_{C,h}$

FCT: $i \in \mathbb{Z}_n$, $X_i \in \mathcal{M}_X$ s.t. $X_i \cong X_{C,h^i}$ ~ powers

ICT: $i \in \mathbb{Z}$, Same thing won't work (by Gauge Theory), so need a generalization of corks

$$(\underbrace{C \subseteq X}_{\text{Noncompact}}, \underbrace{h}_{\text{"End diffeo"}})$$

UCT $X_{i,j} \in \mathcal{M}_{X_i}$



$\Rightarrow \exists$ a single cork (C, h)

s.t. $C \hookrightarrow X_i$ embeds &

$$X_{i,j} = \underbrace{(X_i)_{C, h^j}}_{\text{Enumerates any given list of diffeo classes of a given homeo type}}$$

Proving CT & FCT: Use h-cobordism, some handlebody theory, a "pinwheel" construction