Complete cusp diagram info

What's going on here? Let L be a lattice & $\mathcal{M}(S) := \Gamma \backslash D_S$ be a Hodge-theoretic moduli space of the usual form. Then

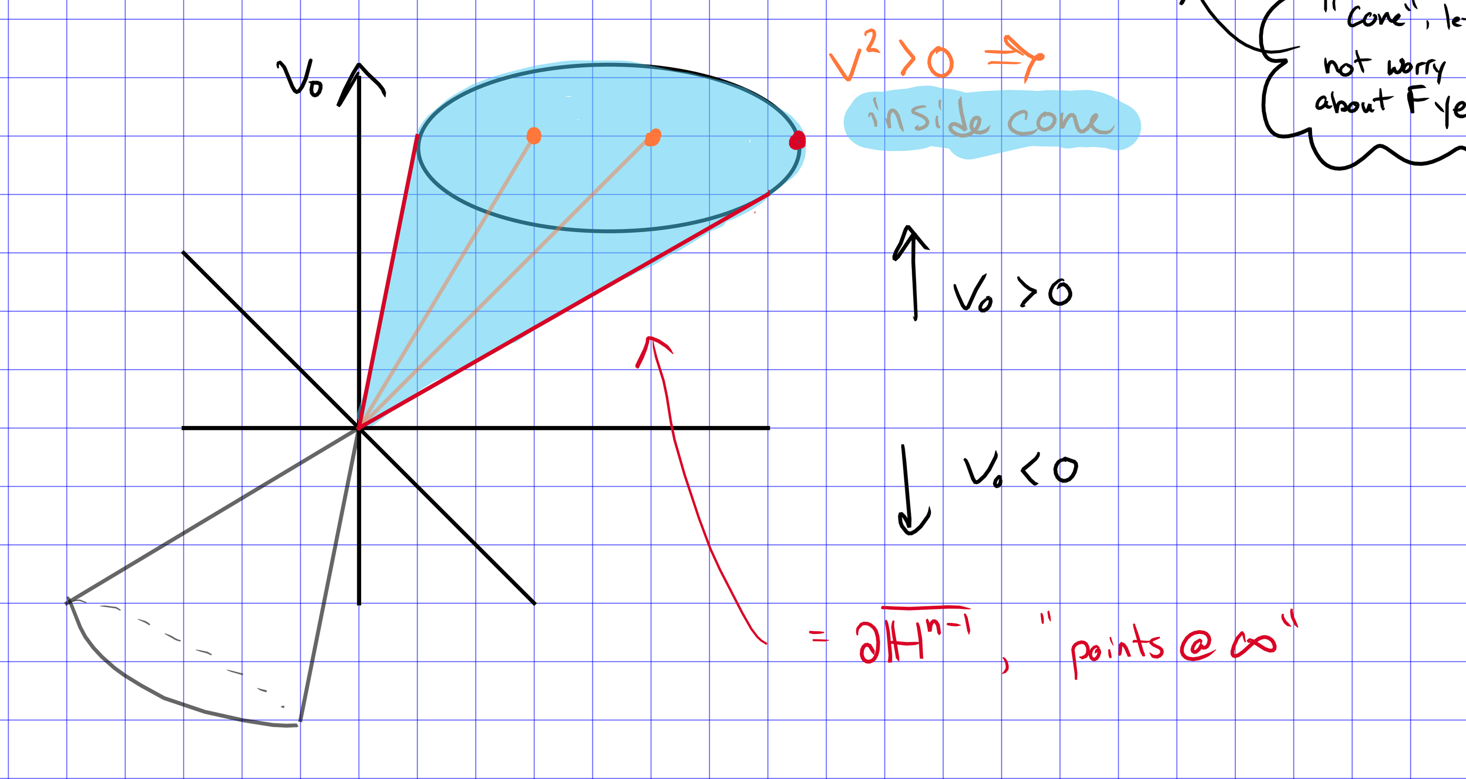
$$\left(\partial \overline{\mathcal{M}(S)}^{BB}\right)_0 := \{0\text{-cusps}\} \cong Gr_1^{iso}(S)/\Gamma = \left\{ \begin{array}{c} \text{classes of isotropic vectors} \\ e \bmod \Gamma \end{array} \right\}$$

$$\left(\partial \overline{\mathcal{M}(S)}^{BB}\right)_1 := \{1\text{-cusps}\} \cong Gr_2^{iso}(S)/\Gamma = \left\{ \begin{array}{c} \text{Classes of isotropic planes} \\ F \bmod \Gamma \end{array} \right\}.$$

If $e \in Gr_1^{iso}(S)$, since $\operatorname{sgn} S = (2, r-2)$ where $r := \operatorname{rk}_{\mathbb{Z}} S$, we have $\operatorname{sgn}(e^\perp/e) = (1, r-3)$ & is thus hyperbolic.

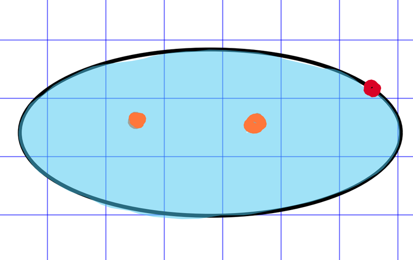
Let $L := e^\perp/e$ and $\operatorname{sgn} L := (1, n)$. We form a Poincaré ball model of $\mathbb{H}^{n-1}$ as $\mathbb{H}_L := \mathbb{P}\{v \in L_{\mathbb{R}} \mid v^2 > 0,\ v_0 > 0\} := C(F)$

"Cone", let's not worry about F yet



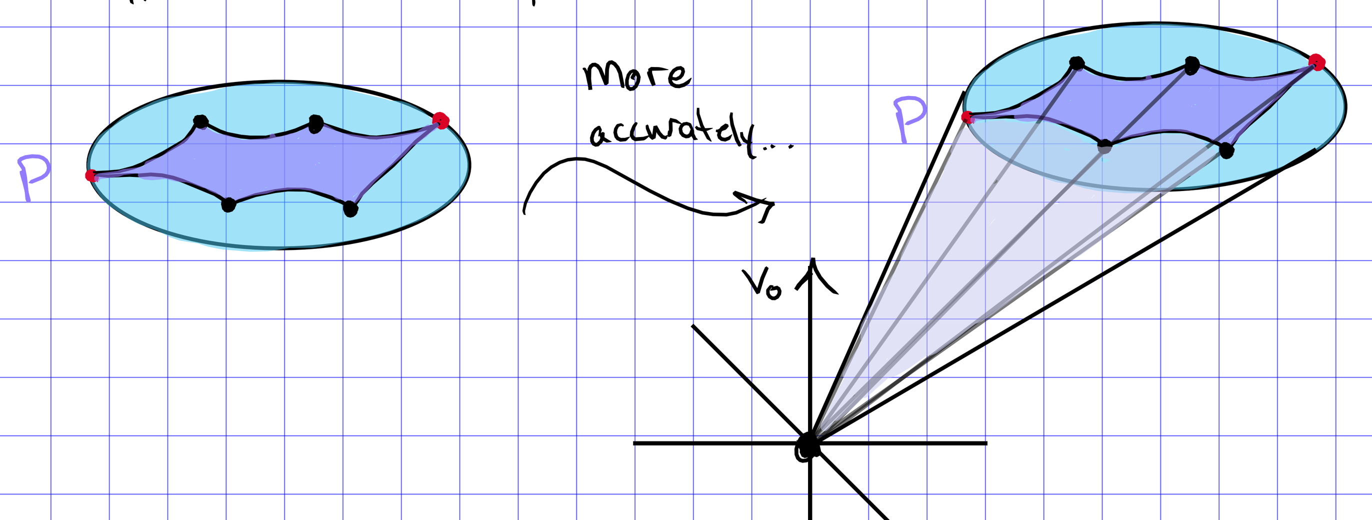
For toroidal compactifications, we want Γ-admissible decompositions of $C(F)$ – lets get some by letting $O(L) \curvearrowright C(F)$ and finding a polyhedral fundamental domain.

So we're thinking about lines in a cone – lets take representatives to be points in the "disc" gotten by slicing the cone with $\{v_0 = 1\}$.

$\overline{\mathbb{H}}^{n-1} =$ 

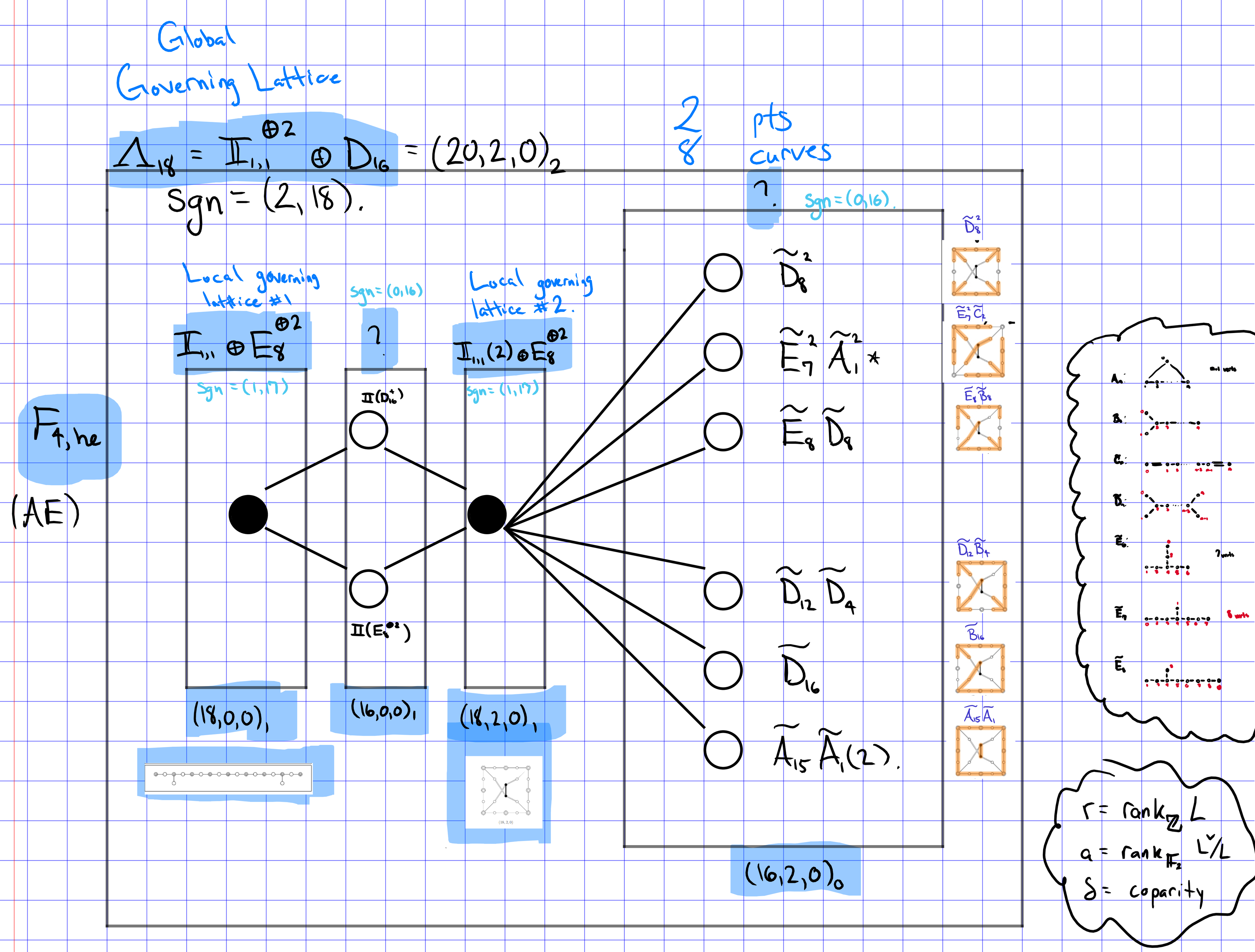
Slight lie: we need $O'(L) \leq O(L)$ the subgroup fixing the cone.

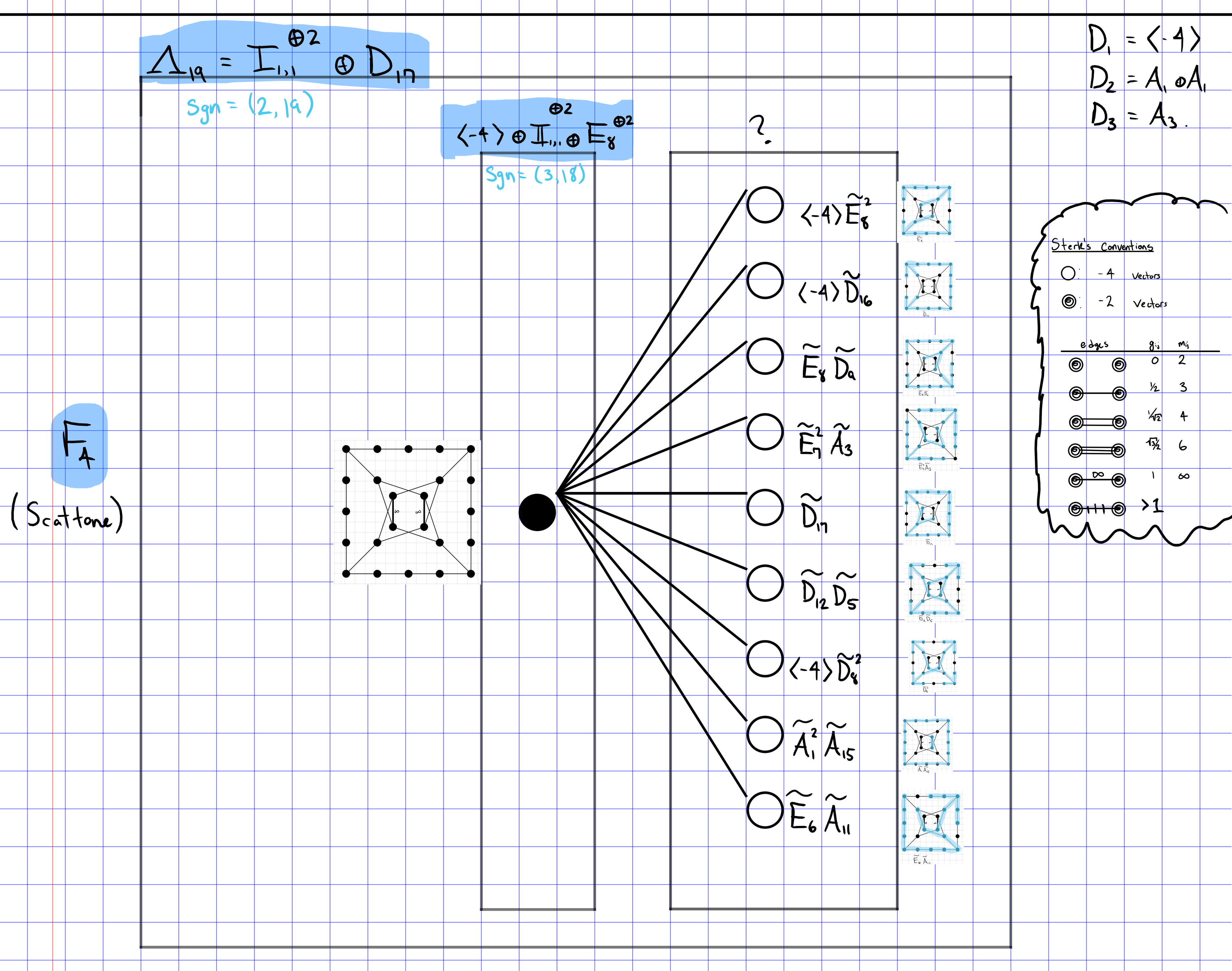
Let's not act by all of $O(L)$, but instead $\Phi_2(L) \leq O(L)$ the reflections in roots, i.e. (-2) vectors. The fundamental domain is a (hyperbolic) Coxeter polytope

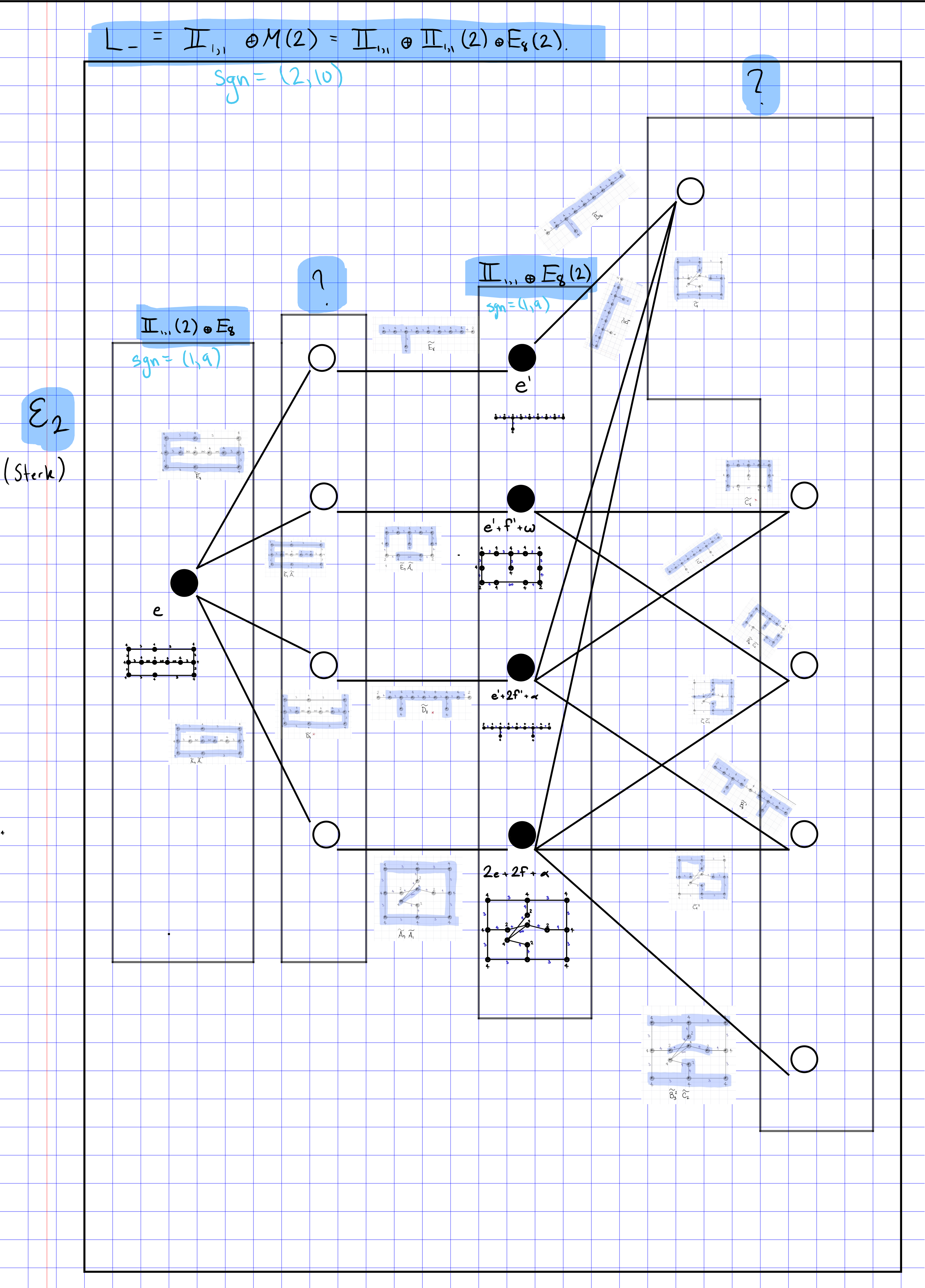


Thus there is a Coxeter diagram D associated to e and its associated 0-cusp. Parabolic subdiagrams of D parameterize incident 1-cusps.

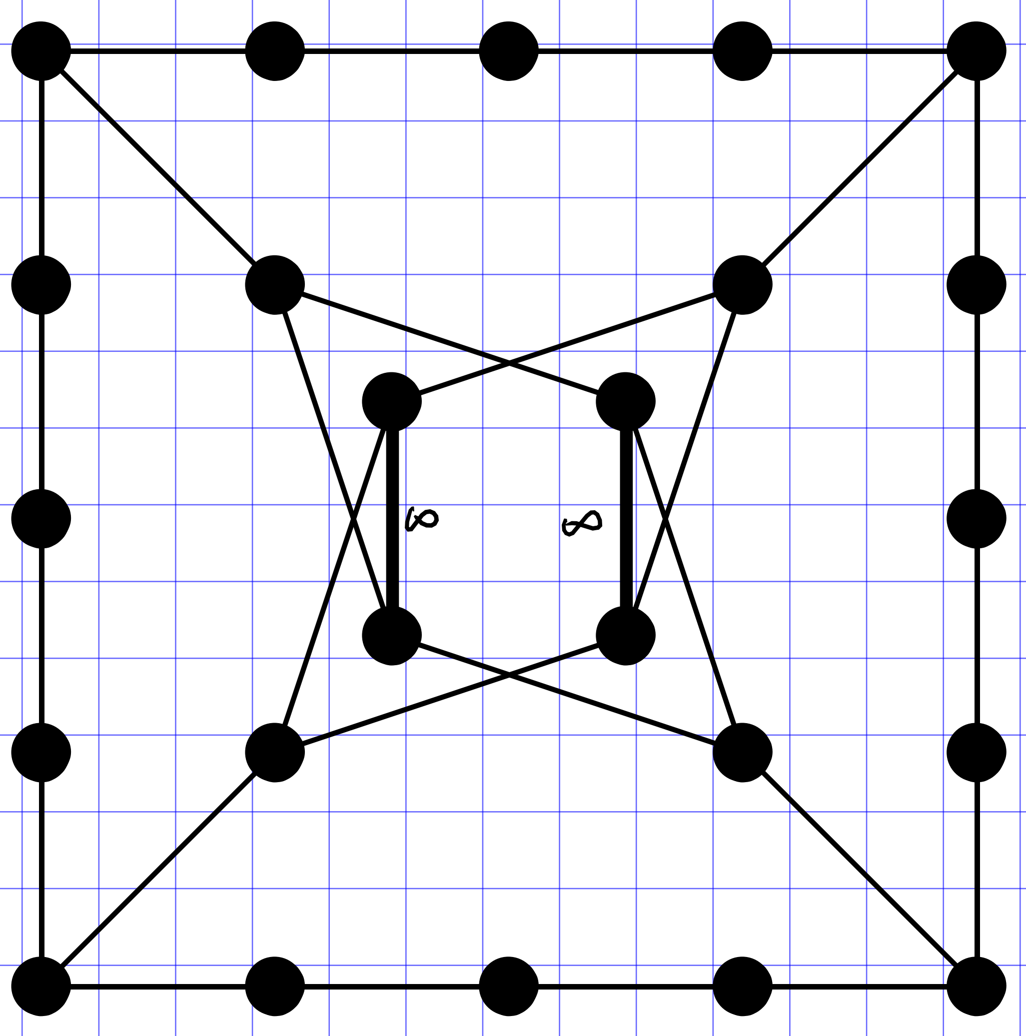
Here: F_4 = K3 deg 4, $F_{4,h.e}$ = hyperelliptic such,
$\mathcal{E}_2$ = polared Enriques deg = 2.

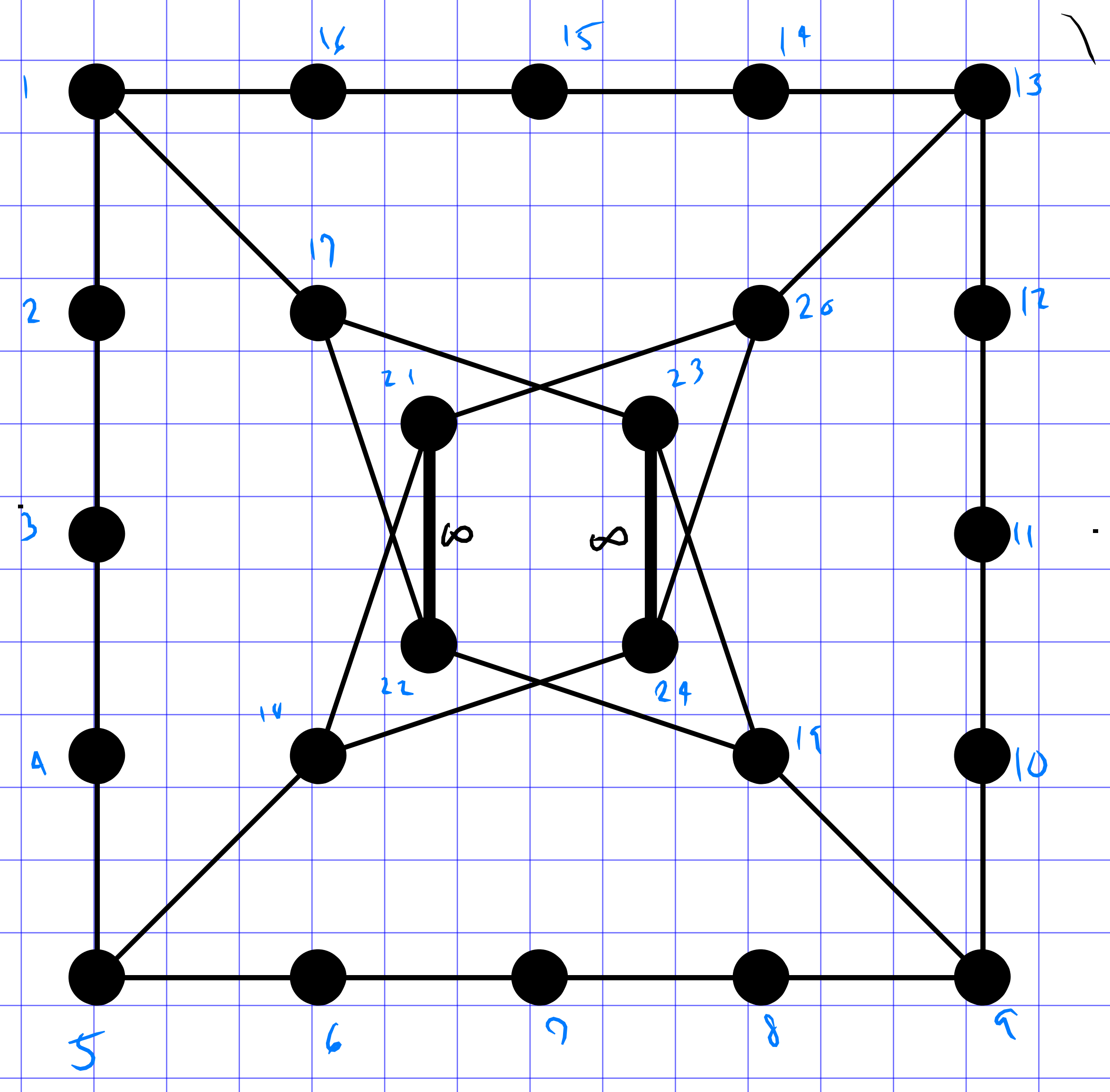


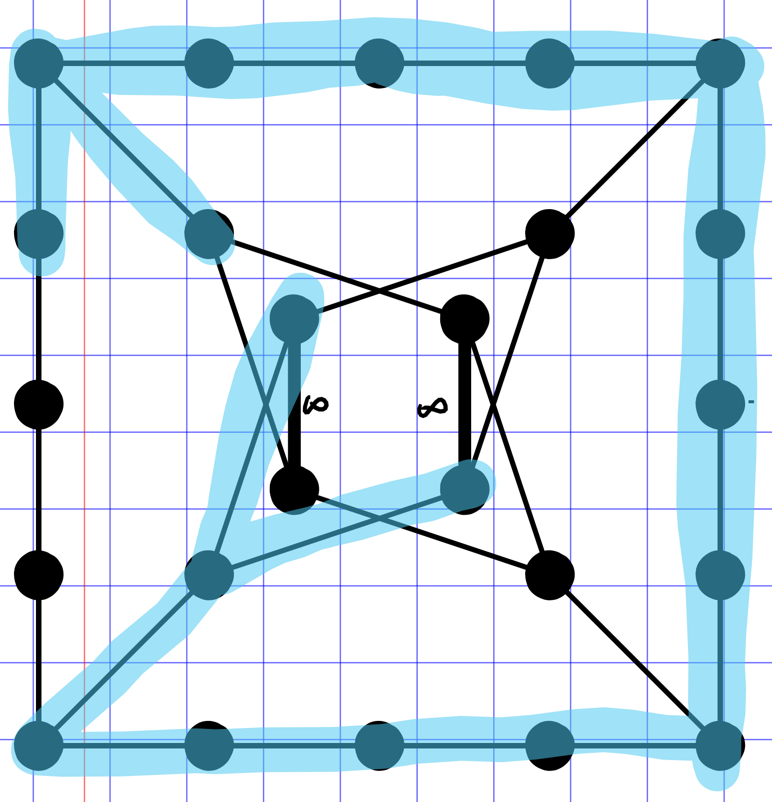




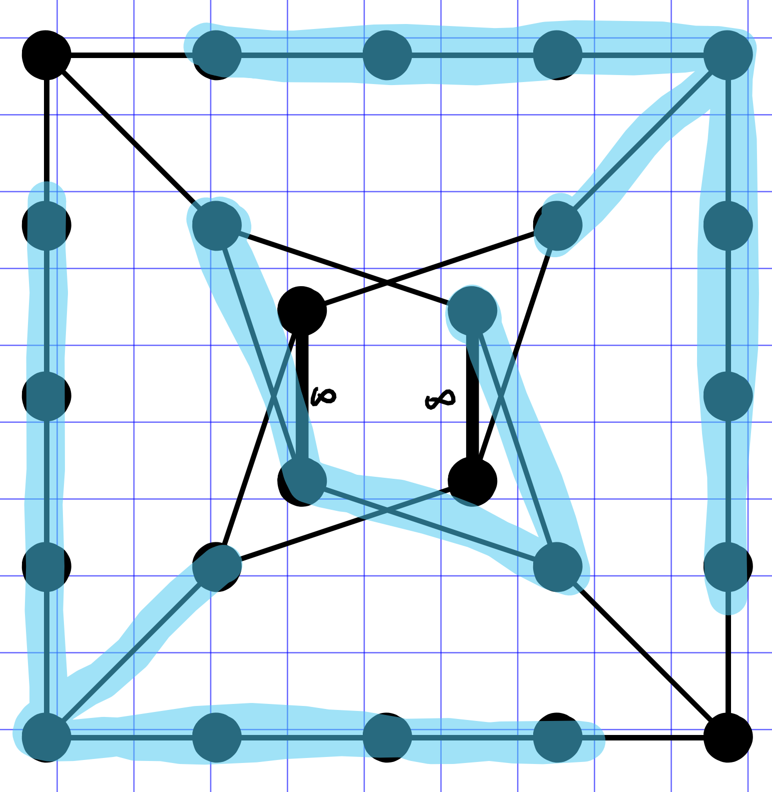
Spotting some parabolics for F_4...



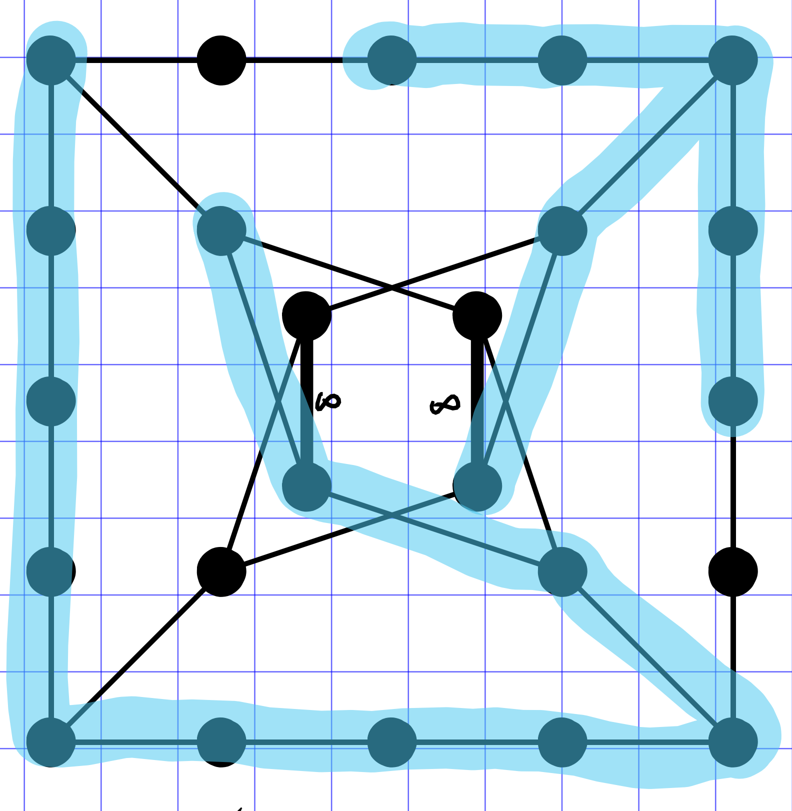




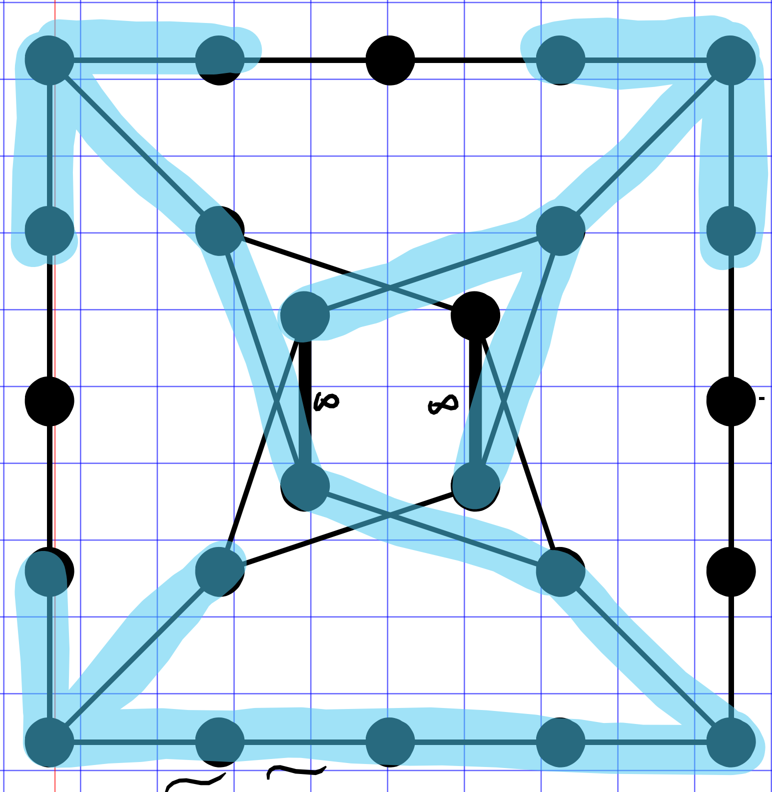
$\widetilde{D}_{17}$



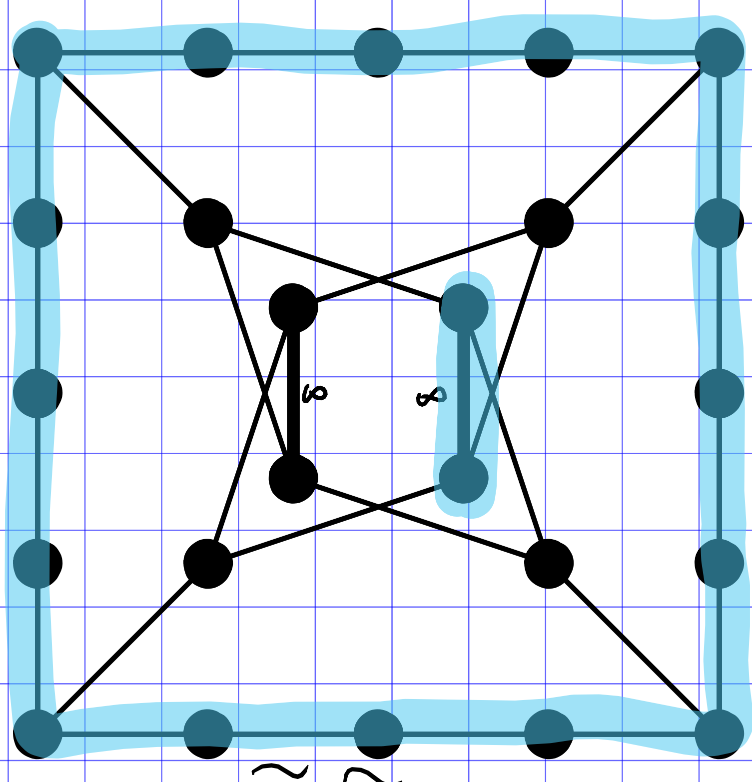
$\widetilde{E}_7^2 \widetilde{A}_3$



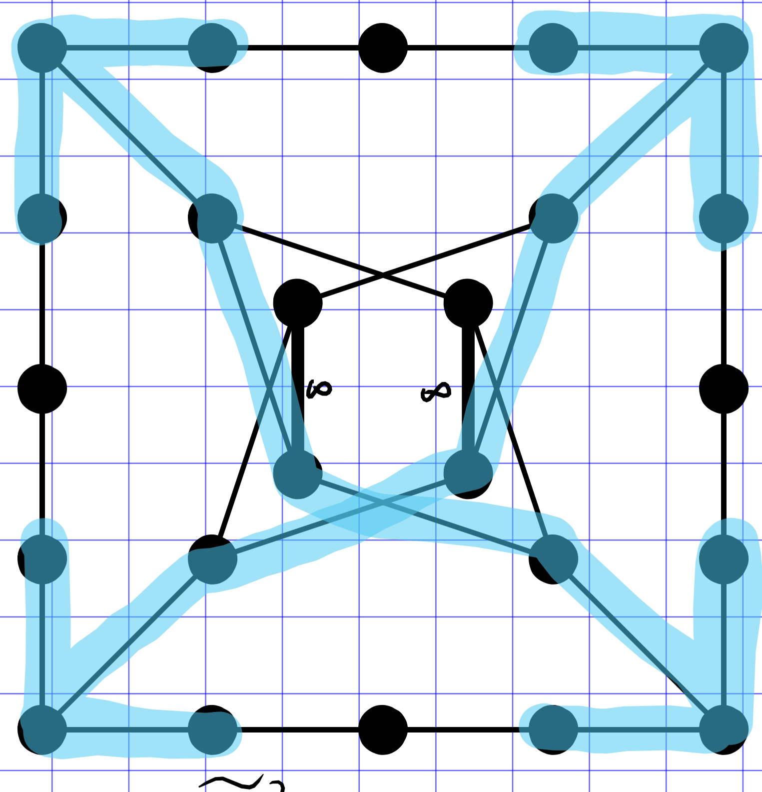
$\widetilde{E}_6 \widetilde{A}_{11}$



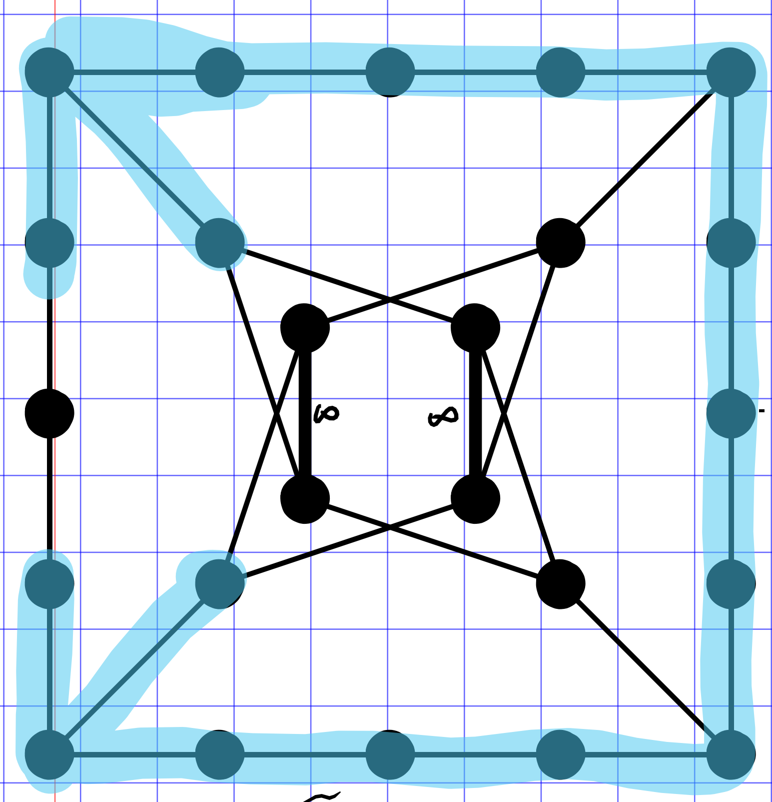
$\widetilde{D}_{12} \widetilde{D}_5$



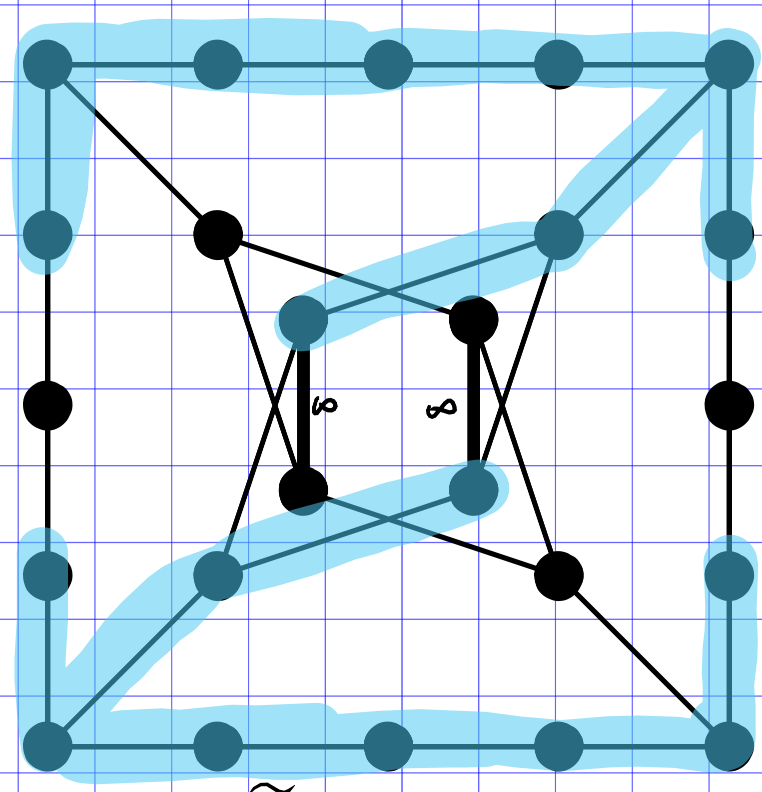
$\widetilde{A}_1 \widetilde{A}_{15}$



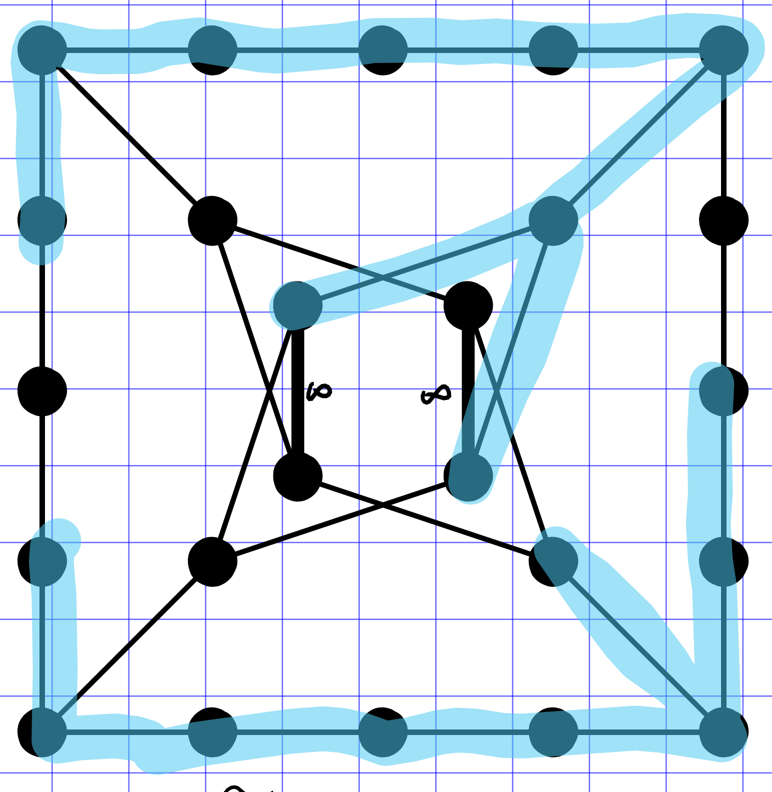
$\widetilde{D}_8^2$



$\widetilde{D}_{16}$



$\widetilde{E}_8^2$



$\widetilde{E}_8 \widetilde{D}_9$